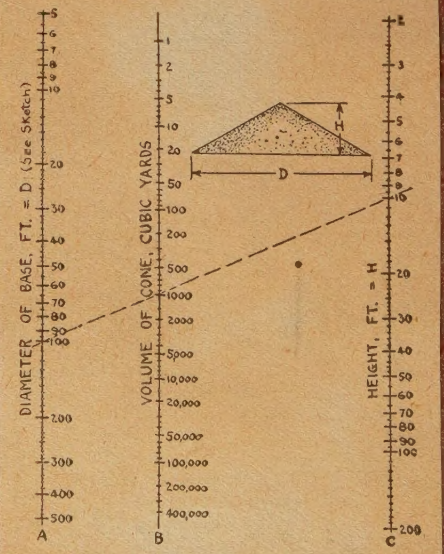
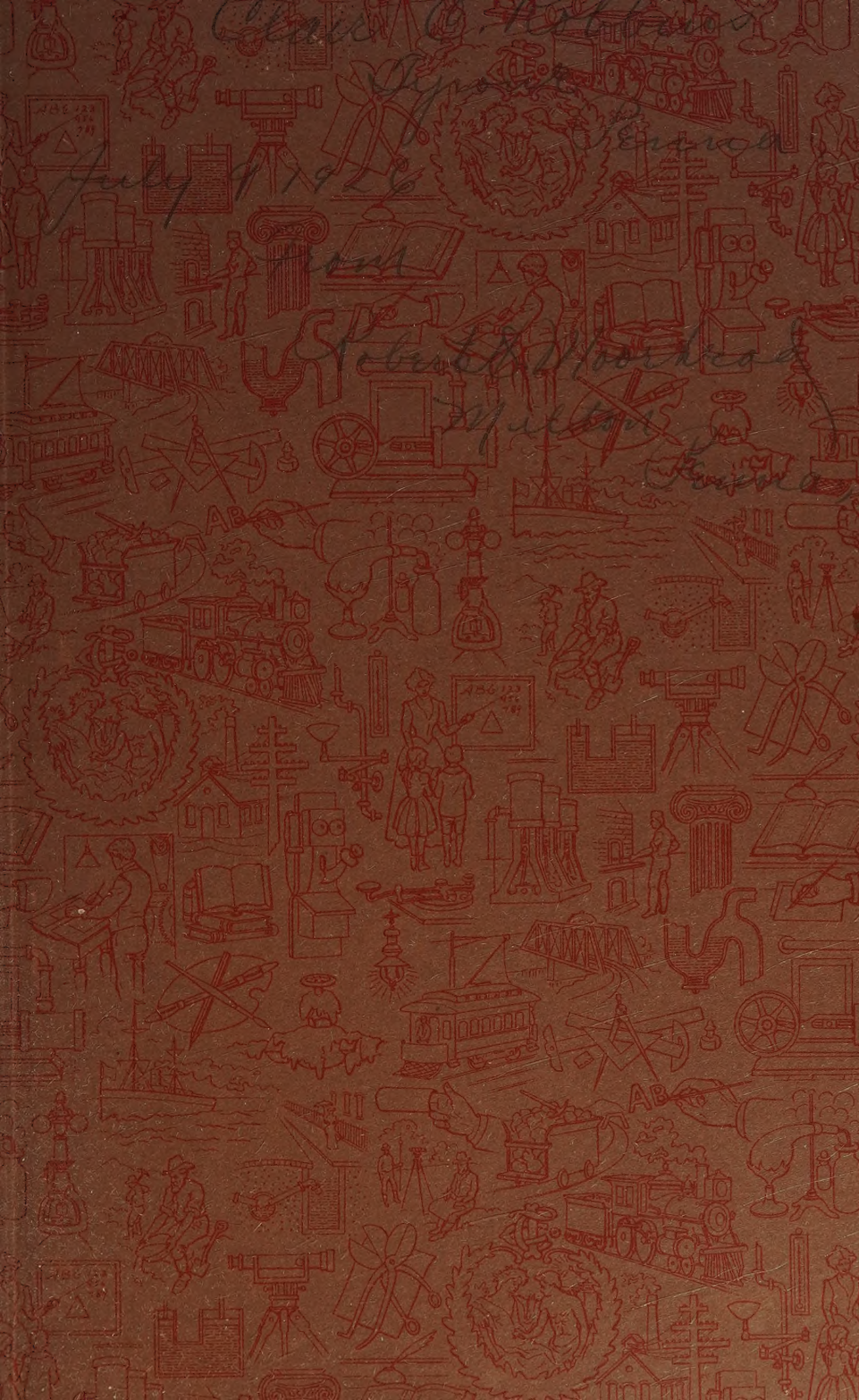
Chart Gives Cubic Yards in Any Conical Pile

WITHOUT ANY figuring whatever this chart quickly gives the number of cubic yards in any conical pile. For example, how many cubic yards in a conical pile whose base diameter is 100 ft. and whose height is 10 ft.? The dotted line drawn across the chart shows how it is done. Run the line through the 100 ft. diameter, column A, and the 10 ft. height, column C, and the intersection with column B gives the answer as a little less than 1000 cubic yards.

The range of the chart takes care of all diameters from 5 to 500 ft. and all heights from 2 to 200 ft. However, in the event that diameters or heights should be greater or less, the same chart is still applicable. Thus if the diameter should be 1000 ft. instead of 100 ft., and if the height should be 100 ft. instead of 10 ft., the same dotted line would solve the problem excepting that the answer would be 1,000,000 cubic yards instead of 1000 cubic yards. In that case the rule is this: For every cipher added to column A, add two ciphers to the answer in column B, and for every cipher added to column C, add



one cipher to the answer in column B. This means that when the problem is 1000 ft. diameter and 100 ft. height, three ciphers must be added to the result in column B.—*W. F. Schaphorst.*



A TEXTBOOK

ON

ARCHITECTURAL DRAWING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ARITHMETIC
FORMULAS
GEOMETRY AND MENSURATION
WITH PRACTICAL QUESTIONS AND EXAMPLES

ANSWERS TO QUESTIONS

7455

SCRANTON
INTERNATIONAL TEXTBOOK CO.
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PREFACE.

The bound volumes of this Course (the Architectural Drawing) have been printed from the plates used in printing the bound volumes of the Complete Architectural Course. The method of numbering the pages, cuts, articles, etc. in the latter named Course was such that each paper and part was complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each paper and part a number. This number was placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it was preceded by the printer's section mark §. The section numbers assigned to the various Instruction Papers incorporated in Vol. I of the present set are as follows: Arithmetic, Part 1, § 1; Arithmetic, Part 2, § 2; Formulas, § 3; Geometry and Mensuration, § 4.

The Question Papers are given the same section numbers as the Instruction Papers having the corresponding title, and are grouped together and placed after the Instruction Papers. The Keys to the Question Papers are grouped together and placed at the end of the volume.

Vol. II contains the Drawing Plates and the instructions for drawing them; it forms a complete course in Architectural Drawing. For convenience the sections have been numbered 1, 2, 3, and 4.

It will be noticed that the indexes to these two volumes

give a double reference—first, to the section; second, to the page. Thus, the references to "Meter," Vol. I, are § 2, page 11. To find it, open the volume and turn the leaves, glancing at the inside edges of the headlines until § 2 is found, and then find page 11.

These volumes will be found very useful by draftsmen in solving problems that so frequently arise in the architect's office, pertaining to the disposition of parts of the plan and elevations, and to the calculation of the quantities of materials involved in the construction.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

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ARITHMETIC.

DEFINITIONS.

1. **Arithmetic** is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* boy, *one* horse, *one* dozen.

3. A **number** is a unit or a collection of units, as *one*, *three* apples, *five* boys.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *horses*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers which express units of the *same kind*, as six *days* and ten *days*, two *feet* and five *feet*.

8. **Unlike numbers** are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *dollars* and five *feet*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading the numbers which have been expressed by figures or letters.

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12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and when standing alone has no value.

The other nine figures are called **digits,** and each has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each figure must have a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local,** or **relative,** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local,** or **relative,** value of a figure is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus 66

The original figure is still *six units*, but the second figure is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousands** 66666

A sixth 6 would be 6 **hundreds of thousands** . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions six hundred sixty-six thousands six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local*, or *relative*, value. Each figure is *ten times* greater in value than the one immediately on its *right.*

19. The **cipher** (0) has no value in itself, but it is useful in determining the place of other figures. To represent the number *four hundred five,* two digits only are necessary, one to represent *four hundred*, and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens.* To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens.* *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds and tens;* thus, 450. *Four thousand and fifty* would be expressed 4050, the first cipher indicating that there are no units and the second that there are no hundreds.

20. In *reading* numbers that have been represented by figures, it is usual to *point off* the number into groups of three figures each, beginning with the right-hand, or **units,** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2 ,	1	9	8 ,	7	6	5 ,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units, tens, hundreds;* the next group of three figures is *thousands;* therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands, tens of thousands, hundreds of thousands,* and insert another comma. We next read *millions, tens of millions, hundreds of millions* (insert another comma), *billions, tens of billions, hundreds of billions.*

The entire line of figures would be read: *four hundred thirty-two billions one hundred ninety-eight millions seven hundred sixty-five thousands four hundred thirty-two.* When we thus *read* a line of figures it is called **numeration,** and if the **numeration** be changed back to *figures,* it is called **notation.**

For instance, the writing of the following figures,

72,584,623,

would be the **notation,** and the **numeration** would be *seventy-two millions five hundred eighty-four thousands six hundred twenty-three.*

21. Note.—It is customary to leave the *s* off the words millions, thousands, etc., in cases like the above, both in speaking and writing; hence, the above would usually be expressed seventy-two million five hundred eighty-four thousand six hundred twenty-three.

22. The four fundamental processes of arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

23. **Addition** is the *process of finding* the *sum of two or more numbers.* The sign of addition is +. It is read *plus,* and means *more.* Thus, $5+6$ is read *5 plus 6,* and means that 5 and 6 are to be added.

24. The sign of equality is =. It is read *equals* or *is equal to.* Thus, $5+6=11$ may be read *5 plus 6 equals 11.*

25. *Like* numbers can be added, but *unlike* numbers cannot be added. Thus, 6 dollars can be added to 7 dollars, and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus 17 and 0 is 17.

27. For addition, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it*, thus bringing units under units, tens under tens, etc. Proceed as in the following examples:

28. Example.—What is the sum of 131, 222, 21, 2, and 413?

Solution.—

$$\begin{array}{r} 131 \\ 222 \\ 21 \\ 2 \\ 413 \\ \hline \textit{sum}\ 789 \end{array}$$ Ans.

Explanation. — After placing the numbers in proper order, begin at the bottom of the right-hand, or units, column, and add, mentally repeating the different sums. Thus, three and two are five and one are six and two are eight and one are nine, the sum of the numbers in units column. Place the 9 directly beneath as the first, or units, figure in the sum.

The sum of the numbers in the next, or tens, column equals 8 tens, which is the second, or tens, figure in the sum.

The sum of the numbers in the next, or hundreds, column equals 7 hundreds, which is the third, or hundreds, figure in the sum.

The sum, or answer, is 789.

29. Example.—What is the sum of 425, 36, 9,215, 4, and 907?

Solution.—

$$\begin{array}{r} 425 \\ 36 \\ 9215 \\ 4 \\ 907 \\ \hline 27 \\ 60 \\ 1500 \\ 9000 \\ \hline \textit{sum}\ 10587 \end{array}$$ Ans.

Explanation.—The sum of the numbers in the first, or

units, column is seven and four are eleven and five are sixteen and six are twenty-two and five are twenty-seven, or 27 units; i. e., two tens and seven units. Write 27 as shown. The sum of the numbers in the second, or tens, column is six tens, or 60. Write 60 underneath 27, as shown. The sum of the numbers in the third, or hundreds. column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column above, the sum of 16 and 6 was required. We know from the table that $6+6=12$; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than 30, since $20+10=30$; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number, and if the result is greater than 9, increase the second figure of the larger number by 1. Thus, $44+7=?$ $4+7=11$; hence, $44+7=51$.

30. The addition may also be performed as follows:

$$\begin{array}{rr} & 425 \\ & 36 \\ & 9215 \\ & 4 \\ & 907 \\ \hline \textit{sum} & 10587 \end{array}$$ Ans.

EXPLANATION.—The sum of the numbers in units column is 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the two tens and add them to the figures in tens column. The sum of the figures in the tens column, plus the 2 tens reserved and carried from the units column, is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next

column. $1 + 9 = 10$, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. Example.—Add the numbers in the column below:

Solution.—

	890
	82
	90
	393
	281
	80
	770
	83
	492
	80
	383
	84
	191
sum	3899 Ans.

Explanation.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write down the 9 and carry 1 to the next column. The sum of the digits in the second column + 1 is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 reserved is 38.

The entire sum is 3,899.

32. **Rule.—I.** *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

33. **Proof.**—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of:

(*a*)	104 + 203 + 613 + 214.	(*a*)	1,134.
(*b*)	1,875 + 3,143 + 5,826 + 10,832.	(*b*)	21,676.
(*c*)	4,865 + 2,145 + 8,173 + 40,084.	(*c*)	55,267.
(*d*)	14,204 + 8,173 + 1,065 + 10,042.	(*d*)	33,484.
(*e*)	10,832 + 4,145 + 3,133 + 5,872.	(*e*)	23,982.
(*f*)	214 + 1,231 + 141 + 5,000.	(*f*)	6,586.
(*g*)	123 + 104 + 425 + 126 + 327.	(*g*)	1,105.
(*h*)	6,354 + 2,145 + 2,042 + 1,111 + 3,333.	(*h*)	14,985.

Ans.

SUBTRACTION.

35. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference,** or **remainder.**

36. The sign of subtraction is −. It is read *minus*, and means *less*. Thus, 12 − 7 is read *12 minus 7*, and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend	7 5 6 8	
subtrahend	3 4 2 5	
remainder	4 1 4 3	Ans.

EXPLANATION.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—

minuend	8 4 5 3	
subtrahend	8 4 4	
remainder	7 6 0 9	Ans.

EXPLANATION.—Begin at the right-hand, or units, column to subtract. We cannot take 4 from 3, and must, therefore, borrow 1 from 5 in tens column and annex it to the 3 in units column. The 1 ten = 10 units, which added to the 3 in units column = 13 units. 4 from 13 = 9, the first, or units, figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second, or tens, figure. We cannot take 8 from 4, and must, therefore, borrow 1 from 8 in thousands column. Since 1 thousand = 10 hundreds, 10 hundreds + 4 hundreds = 14 hundreds, and 8 from 14 = 6, the third, or hundreds, figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, or answer.

The operation of borrowing is performed by mentally placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend	10000
subtrahend	8763
remainder	1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. **Rule.**—*Place the subtrahend (or smaller number) under the minuend (or larger number), in the same manner as for addition, and proceed as in Arts.* **37, 38,** *and* **39.**

41. Proof.—*To prove an example in subtraction, add the subtrahend and the remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000

EXAMPLES FOR PRACTICE.

42. From:

(*a*)	94,278 take 62,574.	(*a*)	31,704.
(*b*)	53,714 take 25,824.	(*b*)	27,890.
(*c*)	71,832 take 58,109.	(*c*)	13,723.
(*d*)	20,804 take 10,408.	(*d*)	10,396.
(*e*)	310,465 take 102,141.	(*e*)	208,324.
(*f*)	(81,043 + 1,041) take 14,831.	(*f*)	67,253.
(*g*)	(20,482 + 18,216) take 21,214.	(*g*)	17,484.
(*h*)	(2,040 + 1,213 + 542) take 3,791.	(*h*)	4.

Ans.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. Multiplication is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. The sign of multiplication is $\times$. It is read *times* or *multiplied by*. Thus, 9×6 is read *9 times 6*, or *9 multiplied by 6*.

46. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

47. In the following table, the product of any two numbers (neither of which exceeds 12) may be found:

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	4 2 5	
multiplier	5	
product	2 1 2 5	Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the right-hand figure of the multiplicand. On looking in the multiplication table, we see that 5×5 are 25. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier, 5, it is seen that 5 times 5 units are 25 units, or 2 tens and 5 units. Write the 5 units in units place in the product, and reserve the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5×2 are 10. Multiplying the second figure of the multiplicand by the multiplier, 5, we see that 5 times 2 tens are 10 tens, and 10 tens plus the 2 tens reserved are 12 tens, or 1 hundred plus 2 tens. Write the 2 tens in tens place, and reserve the 1 hundred to add to the product of hundreds. Again, we see by the multiplication table that 5×4 are 20. Multiplying the third, or last, figure of the multiplicand by the multiplier, 5, we see that 5 times 4 hundreds are 20 hundreds, and 20 hundreds plus the 1 hundred reserved are 21 hundreds, or 2 thousands and 1 hundred, which we write in thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

	4 2 5
	4 2 5
	4 2 5
	4 2 5
	4 2 5
sum	2 1 2 5 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of:

(*a*) $61{,}483 \times 6$.
(*b*) $12{,}375 \times 5$.
(*c*) $10{,}426 \times 7$.
(*d*) $10{,}835 \times 3$.

Ans. (*a*) 368,898. (*b*) 61,875. (*c*) 72,982. (*d*) 32,505.

(*e*)	98,376 × 4.	Ans.	(*e*) 393,504.
(*f*)	10,873 × 8.		(*f*) 86,984.
(*g*)	71,543 × 9.		(*g*) 643,887.
(*h*)	218,734 × 2.		(*h*) 437,468.

50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

multiplicand	475
multiplier	234
	1900
	1425
	950
product	111150 Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial product;* 3 times 475 = 1,425, the *second partial product, the right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. **Proof.**—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply by it the same as with the other figures. Thus,

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
× 0	× 0	× 0	× 0
0 Ans.	0 Ans.	00 Ans.	000 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
0000	0000	00000
6228	12024	00000
632142 Ans.	1222440 Ans.	31264
		31326528 Ans.

When multiplying by a number containing a cipher, the work may be shortened by writing the first cipher of the partial product, then multiplying by the next figure of the multiplier and writing the partial product alongside of the cipher. Thus, examples (*e*) and (*g*) above might have been solved in the following manner:

3114	31264
203	1002
9342	62528
62280	3126400
632142 Ans.	31326528 Ans.

EXAMPLES FOR PRACTICE.

54. Find the product of:

(*a*) 3,842 × 26.
(*b*) 3,716 × 45.
(*c*) 1,817 × 124.
(*d*) 675 × 38.
(*e*) 1,875 × 33.
(*f*) 4,836 × 47.
(*g*) 5,682 × 543.
(*h*) 3,257 × 246.

Ans. (*a*) 99,892. (*b*) 167,220. (*c*) 225,308. (*d*) 25,650. (*e*) 61,875. (*f*) 227,292. (*g*) 3,085,326. (*h*) 801,222.

(*i*)	$2{,}875 \times 302$.		(*i*) 868,250.
(*j*)	$17{,}819 \times 1{,}004$.		(*j*) 17,890,276.
(*k*)	$38{,}674 \times 205$.		(*k*) 7,928,170.
(*l*)	$18{,}304 \times 100$.	Ans.	(*l*) 1,830,400.
(*m*)	$7{,}834 \times 10$.		(*m*) 78,340
(*n*)	$87{,}543 \times 1{,}000$.		(*n*) 87,543,000.
(*o*)	$48{,}763 \times 100$.		(*o*) 4,876,300.

DIVISION.

55. **Division** is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which *shows* how many times the *divisor is contained in the dividend* is called the **quotient.**

56. The sign of division is $\div$. It is read *divided by.* $54 \div 9$ is read *54 divided by 9.* Another way to write 54 divided by 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$.

In both of these cases, 54 is the *dividend* and 9 is the *divisor.*

Division is the reverse of multiplication.

57. **To divide when the divisor consists of but one figure,** proceed as in the following example:

EXAMPLE.—What is the quotient of $875 \div 7$?

SOLUTION.—

divisor dividend quotient

$$7)875(125 \text{ Ans.}$$
$$7$$
$$17$$
$$14$$
$$35$$
$$35$$
remainder $$0$$

EXPLANATION.— 7 is contained in 8 hundreds, 1 hundred times. Place the 1 as the first, or left-hand, figure of the quotient. Multiply the divisor, 7, by the 1 hundred of the

quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. Beside the remainder, 1, bring down the next, or tens, figure of the dividend, in this case 7, making 17 tens; 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor, 7, by the 2 in the quotient, and subtract the product from 17. Beside the remainder, 3, bring down the units figure of the dividend, making 35 units. 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend
divisor 7)$8^{1}7^{3}5$
quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, once and 1 remainder; imagine 1 to be placed before 7, making 17; 7 is contained in 17, 2 times and 3 over; imagine 3 to be placed before 5, making 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient 125.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

divisor *dividend* *quotient*

SOLUTION.— 63) 2702826 (42902 Ans.

252

182

126

568

567

126

126

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270. 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first, or left-hand, figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product, 252, from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product, 189, is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor, 63, by 2 and subtracting the product, 126, from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568. 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9 and subtract the product, 567, from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

60. **Rule.—I.** *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be at last a remainder, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor and add the remainder, if there be any, to the product. The result will be the dividend.* Thus,

divisor *dividend* *quotient*

63) 4235 (67$\frac{14}{63}$ Ans.

378

455

441

remainder 14

PROOF.— *quotient* 67

divisor 63

201

402

4221

remainder 14

dividend 4235

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*)	126,498 by 58.	(*a*)	2,181.
(*b*)	3,207,594 by 767.	(*b*)	4,182.
(*c*)	11,408,202 by 234.	(*c*)	48,753.
(*d*)	2,100,315 by 581.	(*d*)	3,615.
(*e*)	969,936 by 4,008.	(*e*)	242.
(*f*)	7,481,888 by 1,021.	(*f*)	7,328.
(*g*)	1,525,915 by 5,003.	(*g*)	305.
(*h*)	1,646,301 by 381.	(*h*)	4,321.

Ans.

CANCELATION.

63. Cancelation is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers, which, when multiplied together, *produce the given number*. Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which cannot be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc. are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

The canceling of a factor in *both* dividend and divisor is the same as *dividing them both by the same number*, and this, evidently, *does not change the quotient.*

Write the numbers forming the dividend above a horizontal line, and those forming the divisor below it; then cancel the equal factors.

68. Example.—Divide $4 \times 45 \times 60$ by 9×24.

Solution.—Placing the dividend over the divisor, and canceling,

$$\frac{\overset{5}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{\underset{6}{\cancel{9}} \times \cancel{24}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

Explanation.—The 4 in the dividend and the 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the 4 and write the 1 over it; also, cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also, cross off the 6 and write 1 under it. Thus,

$$\frac{1 \qquad \overset{10}{\cancel{60}}}{4 \times 45 \times \cancel{60}} =$$
$$\underset{1}{\cancel{6}}$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also, cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times 60}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

Hence, $\dfrac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{6}{\cancel{24}}} = \dfrac{1 \times 5 \times 10}{1 \times 1} = 50.$ Ans.

69. **Rule.—I.** *Cancel the common factors from both the dividend and the divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide:

(*a*)	$14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.	Ans.	(*a*) 32.
(*b*)	$3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.		(*b*) 250.
(*c*)	$8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.		(*c*) 1.
(*d*)	$164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.		(*d*) 48.
(*e*)	$50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.		(*e*) 5.
(*f*)	$48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.		(*f*) 105.
(*g*)	$110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.		(*g*) 42.
(*h*)	$115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.		(*h*) 5.

FRACTIONS.

DEFINITIONS.

71. A **fraction** is a *part of a unit. One-half, one-third, two-fifths* are fractions.

72. *Two* numbers are required to express a fraction; one is called the **numerator,** and the other, the **denominator.**

73. The numerator is placed above the denominator, with a line between them, as $\frac{2}{3}$. Here, 3 is the denominator, and shows into how many *equal parts* the *unit*, or *one*, is divided. The numerator, 2, shows how many of these equal parts are taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine forty-sevenths.

74. In the expression "$\frac{3}{4}$ of an apple," the *denominator*, 4, shows that the apple is to be (or has been) cut into 4 *equal parts*, and the *numerator*, 3, shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the *parts*, or *fourths*, of the apple were cut into *two equal pieces*, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all, one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, and written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc. are abbreviations of three one-fourths, three one-eighths, five one-sixteenths, etc. It is evident that the larger the *denominator*, the greater is the number of parts into which anything is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is smaller than $\frac{7}{8}$, because if an object be divided into 9 parts, the parts are smaller than if

the same object had been divided into 8 parts; and, since $\frac{1}{9}$ is smaller than $\frac{1}{8}$, it is clear that 7 one-ninths is a smaller amount than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

75. The **value** of a fraction is the numerator divided by the denominator, as $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

76. The line between the numerator and the denominator means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.

$\frac{5}{8}$ is equivalent to $5 \div 8$.

77. The numerator and the denominator of a fraction are called the **terms** of a fraction.

78. The *value* of a fraction whose numerator and denominator are equal is 1.

$\frac{4}{4}$, or four-fourths $= 1$.

$\frac{8}{8}$, or eight-eighths $= 1$.

$\frac{64}{64}$, or sixty-four sixty-fourths $= 1$.

79. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

80. An **improper fraction** is a fraction whose numerator *equals* or is *greater* than the denominator. Its value is 1 or *more than* 1, as $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

81. A **mixed number** is a whole number and a fraction united. $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

82. **Reduction of fractions** is the process of changing their form without changing their *value.*

83. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The value is not changed, since $\frac{3}{4} = \frac{6}{8}$. For, suppose that an object, say an apple, is divided into 8 equal parts. If these parts be arranged into 4 piles, each containing 2 parts, it is evident each pile will be composed of the same amount of the entire apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one-quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same course of reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

84. To reduce a fraction to an equal fraction having a given denominator:

EXAMPLE.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will, in this case, make the product 96; this number is evidently $96 \div 8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Ans.

85. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the result.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

86. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in Art. **83**; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

87. A fraction is reduced to its *lowest terms* when its numerator and denominator cannot both be divided by the *same* number without a remainder; for example, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{24}$, $\frac{8}{15}$.

EXAMPLES FOR PRACTICE.

88. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.
(*b*) $\frac{24}{132}$ to its lowest terms.
(*c*) $\frac{64}{1000}$ to its lowest terms.
(*d*) $\frac{6}{7}$ to 49ths.
(*e*) $\frac{13}{16}$ to 10,000ths.

Ans. (*a*) $\frac{56}{128}$. (*b*) $\frac{2}{11}$. (*c*) $\frac{8}{125}$. (*d*) $\frac{35}{49}$. (*e*) $\frac{8125}{10000}$.

89. To reduce a whole number or a mixed number to an improper fraction:

EXAMPLE.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.— $8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

90. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

EXAMPLES FOR PRACTICE.

91. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{1}{9}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{8}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. (*a*) $\frac{33}{8}$. (*b*) $\frac{46}{9}$. (*c*) $\frac{102}{10}$. (*d*) $\frac{151}{4}$. (*e*) $\frac{254}{6}$. (*f*) $\frac{112}{16}$.

92. To reduce an improper fraction to a whole or a mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times and 1 remaining (see Art. 75); as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

93. **Rule.**—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

94. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.
(*b*) $\frac{185}{8}$.
(*c*) $\frac{701}{6}$.
(*d*) $\frac{149}{3}$.
(*e*) $\frac{76}{19}$.
(*f*) $\frac{125}{25}$.

Ans. (*a*) $24\frac{1}{6}$. (*b*) $61\frac{2}{8}$. (*c*) $116\frac{5}{6}$. (*d*) $49\frac{2}{3}$. (*e*) 4. (*f*) 5.

---

95. A **common denominator** of two or more fractions is a number which will contain (i. e., which may be divided by) the denominator of each of the given fractions without a remainder. The **least common denominator** is the least number that will contain each denominator of the given fractions without a remainder.

96. **To find the least common denominator:**

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

2) 4, 3, 9, 16
2) 2, 3, 9, 8
3) 1, 3, 9, 4
3) 1, 1, 3, 4
4) 1, 1, 1, 4
1, 1, 1, 1

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide each of them by some prime number which will divide at least two of them without a remainder (if possible), bringing down those denominators to the row below which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4. Dividing the third row by 3, the result, is 1, 1,

3, 4. So continue until the last row contains only 1's. The product of all the divisors, or $2 \times 2 \times 3 \times 3 \times 4 = 144$, is the least common denominator.

97. EXAMPLE.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r|l} 3 & 9,\ 12,\ 18 \\ \hline 3 & 3,\ 4,\ 6 \\ \hline 2 & 1,\ 4,\ 2 \\ \hline 2 & 1,\ 2,\ 1 \\ \hline & 1,\ 1,\ 1 \end{array}$$

$3 \times 3 \times 2 \times 2 = 36$. Ans.

98. To reduce two or more fractions to fractions having a common denominator:

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

SOLUTION.—The common denominator is a number which will contain 3, 4, and 2. The least common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder.

$$\frac{2}{3} = \frac{8}{12}, \quad \frac{3}{4} = \frac{9}{12}, \quad \frac{1}{2} = \frac{6}{12}.$$

Reducing $\frac{2}{3}$ (see Art. 84), 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. In the same way we find $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

99. Rule.—*Divide the common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

100. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.
(*d*) $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.
(*f*) $\frac{7}{15}$, $\frac{17}{30}$, $\frac{21}{30}$.

Ans.

(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$.

ADDITION OF FRACTIONS.

101. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into 2 equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now, if we add these parts, the result is $2+4=6$ something. But what is this something? It is not fourths, for 6 fourths are $1\frac{1}{2}$, and we had only 1 apple to begin with; neither is it eighths, for 6 eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the quarters to eighths, we have $\frac{2}{4}=\frac{4}{8}$, and adding the other 4 eighths, $4+4=8$ eighths. This result is correct, since $\frac{8}{8}=1$. Or we can, in this case, reduce the eighths to quarters. Thus, $\frac{4}{8}=\frac{2}{4}$; whence, adding, $2+2=4$ quarters, a correct result, since $\frac{4}{4}=1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

102. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\frac{1}{2}=\frac{4}{8},\quad \frac{3}{4}=\frac{6}{8},\quad \text{and}\quad \frac{5}{8}=\frac{5}{8}.$$

EXPLANATION.—As the denominator tells or indicates the *names* of the parts, the numerators only are added, to obtain the total number of *parts* indicated by the denominator. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths =

$$\frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{4+6+5}{8}=\frac{15}{8}=1\tfrac{7}{8}.\quad \text{Ans.}$$

103. EXAMPLE.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$12\tfrac{3}{4}=12\tfrac{12}{16}$$
$$14\tfrac{5}{8}=14\tfrac{10}{16}$$
$$7\tfrac{5}{16}=7\tfrac{5}{16}$$
$$\textit{sum}\quad 33+\tfrac{27}{16}=33+1\tfrac{11}{16}=34\tfrac{11}{16}.\quad \text{Ans.}$$

The sum of the fractions $=\frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $=34\frac{11}{16}$.

EXAMPLE.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16}=13\frac{6}{32}$, $3\frac{1}{4}=3\frac{8}{32}$.

$$17$$
$$13\tfrac{6}{32}$$
$$\tfrac{9}{32}$$
$$3\tfrac{8}{32}$$
$$\textit{sum}\quad 33\tfrac{23}{32}.\quad \text{Ans.}$$

104. **Rule.—I.** *Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

105. Find the sum of:

(*a*) $\frac{4}{6}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{2}{3}$, $\frac{5}{15}$, $\frac{24}{45}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $\frac{14}{72}$, $\frac{18}{18}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{22}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{3}{7}$.

Ans. (*a*) $1\frac{7}{12}$. (*b*) $1\frac{8}{15}$. (*c*) $1\frac{3}{16}$. (*d*) $1\frac{51}{72}$. (*e*) $1\frac{29}{66}$. (*f*) $1\frac{5}{9}$. (*g*) $1\frac{7}{22}$. (*h*) 1.

SUBTRACTION OF FRACTIONS.

106. *Fractions cannot be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions.

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13-6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

107. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, since $7 = 6+1$, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$, or $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

108. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{5}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{5}{32}$
difference	$8\frac{13}{32}$. Ans.

109. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or $8\frac{20}{16}$	
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
difference	$4\frac{13}{16}$	$4\frac{13}{16}$. Ans.

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

110. EXAMPLE.—From 9 take $8\frac{3}{16}$.

SOLUTION.—

minuend	9	or $8\frac{16}{16}$
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$
difference	$\frac{13}{16}$	$\frac{13}{16}$. Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

111. **Rule.—I.** *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

112. Subtract:

(*a*) $\frac{10}{24}$ from $\frac{11}{12}$.
(*b*) $\frac{7}{14}$ from $\frac{17}{28}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.
(*d*) $\frac{15}{35}$ from $\frac{45}{70}$.
(*e*) $\frac{15}{16}$ from $\frac{57}{48}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(*g*) $12\frac{1}{8}$ from 27.
(*h*) $5\frac{1}{4}$ from 30.

Ans. (*a*) $\frac{1}{6}$. (*b*) $\frac{3}{28}$. (*c*) $\frac{11}{30}$. (*d*) $\frac{3}{14}$. (*e*) $\frac{1}{2}$. (*f*) $17\frac{1}{2}$. (*g*) $14\frac{7}{8}$. (*h*) $24\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS.

113. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

114. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3}{4} \times 4 = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word "of," when placed between two fractions, or between a fraction and a whole number, means the same as $\times$, or times. Thus,

$$\frac{3}{4} \text{ of } 4 = \frac{3}{4} \times 4 = 3.$$

$$\frac{1}{8} \text{ of } \frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}.$$

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$. Ans.

Or, $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

115. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\cancel{4} \times 7}{\cancel{16} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

116. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \cancel{32}} = \frac{3}{8 \times 2} = \frac{3}{16}$. Ans.

117. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39 \times 45}{4 \times 8} = \frac{1755}{32} = 54\frac{27}{32}. \quad \text{Ans.}$$

118. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—

$$\begin{array}{r} 15\frac{7}{8} \\ \underline{\quad 3 \quad} \\ 47\frac{5}{8} \end{array} \quad \text{or} \quad \begin{array}{r} 15\frac{7}{8} \\ \underline{\quad 3 \quad} \\ 45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}. \end{array} \quad \text{Ans.}$$

119. **Rule.—I.** *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number and add the whole-number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

120. Find the product of:

(*a*) $7 \times \frac{3}{19}$.
(*b*) $14 \times \frac{5}{16}$.
(*c*) $\frac{21}{32} \times \frac{5}{14}$.
(*d*) $\frac{16}{27} \times 4$.
(*e*) $\frac{10}{18} \times 7$.
(*f*) $17\frac{18}{21} \times 7$.
(*g*) $\frac{105}{224} \times 32$.
(*h*) $\frac{15}{28} \times 14$.

Ans. (*a*) $1\frac{2}{19}$. (*b*) $4\frac{3}{8}$. (*c*) $\frac{15}{64}$. (*d*) $2\frac{10}{27}$. (*e*) $7\frac{7}{18}$. (*f*) 125. (*g*) 15. (*h*) $7\frac{1}{2}$.

DIVISION OF FRACTIONS.

121. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

122. *Dividing the numerator or multiplying the denominator divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing the numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \quad \text{Ans.}$$

When *multiplying the denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $$\tfrac{3}{16} \div 2 = \frac{3}{16 \times 2} = \tfrac{3}{32}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $$\tfrac{14}{32} \div 7 = \frac{14 \div 7}{32} = \tfrac{2}{32} = \tfrac{1}{16}. \quad \text{Ans.}$$

123. To **invert** a fraction is to *turn it upside down;* that is, make the numerator and denominator *change places.*

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

124. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor, $\frac{3}{16}$, and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This brings the same quotient as in the first case.

125. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

SOLUTION.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\tfrac{3}{8} \div \tfrac{1}{4} = \tfrac{3}{8} \times \tfrac{4}{1} = \frac{3 \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

126. EXAMPLE.—Divide 5 by $\frac{10}{16}$.

SOLUTION.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\overset{8}{\cancel{5}} \times \cancel{16}}{\underset{2}{\cancel{10}}} = 8. \quad \text{Ans.}$$

127. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

128. Rule.—*Invert the divisor and proceed as in multiplication.*

129. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\frac{3\times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\frac{8+4}{16}$.

$\frac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4}\div\frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line is to be divided by all below it.*

EXAMPLES FOR PRACTICE.

130. Divide:

(*a*) 15 by $6\frac{3}{7}$.
(*b*) 30 by $\frac{6}{8}$.
(*c*) 172 by $\frac{4}{5}$.
(*d*) $\frac{14}{18}$ by $1\frac{7}{18}$.
(*e*) $\frac{103}{6}$ by $14\frac{2}{3}$.
(*f*) $\frac{142}{27}$ by $17\frac{1}{3}$.
(*g*) $\frac{14}{18}$ by $\frac{145}{72}$.
(*h*) $\frac{128}{18}$ by $72\frac{1}{3}$.

Ans. (*a*) $2\frac{1}{3}$. (*b*) 40. (*c*) 215. (*d*) $\frac{112}{207}$. (*e*) $11\frac{5}{88}$. (*f*) $\frac{71}{281}$. (*g*) $\frac{56}{145}$. (*h*) $\frac{64}{651}$.

131. Whenever an expression like one of the three following ones is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\frac{\frac{3}{4}}{9} = \frac{3}{9\times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4}\times 4}{9\times 4} = \frac{3}{9\times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9\times4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5\times4}{3\times9} = \frac{20}{27}$. For, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\frac{\frac{5}{9}\times9}{\frac{3}{4}\times9} = \frac{5}{\frac{3\times9}{4}}$; and $\frac{5}{\frac{3\times9}{4}}\times4 = \frac{5\times4}{3\times9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\frac{\frac{1}{4}\times310\times\frac{27}{12}\times72}{40\times4\frac{1}{2}\times5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4}\times310\times\frac{27}{12}\times72}{40\times\frac{9}{2}\times\frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1\times310\times27\times72\times2\times6}{40\times9\times31\times4\times12} = \frac{1\times\cancel{310}^{10}\times\cancel{27}^{3}\times\cancel{72}^{6}\times\cancel{2}\times\cancel{6}^{3}}{\cancel{40}_{4}\times\cancel{9}\times\cancel{31}\times\cancel{4}_{2}\times\cancel{12}_{2}}$$

$$= \frac{27}{2} = 13\tfrac{1}{2}.$$

Greater exactness in results can usually be obtained by using this principle than by reducing the fractions to decimals. The principle, however, should not be employed if a sign of addition or subtraction occurs either above or below the dividing line.

DECIMALS.

132. **Decimals** are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

133. The *denominator* which is always ten or a multiple of ten, as 10, 100, 1,000, etc., is not expressed, as it would

be in common fractions, by writing it under the *numerator* with a line between them, as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, but is expressed by placing a *period* (.), which is called a **decimal point**, to the *left* of the *figures of the numerator*, so as to indicate that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.

134. The *reading* of a decimal number depends upon the number of decimal places in it, or the number of figures to the right of the decimal point.

One decimal place expresses *tenths.*
Two decimal places express *hundredths.*
Three decimal places express *thousandths.*
Four decimal places express *ten-thousandths.*
Five decimal places express *hundred-thousandths.*
Six decimal places express *millionths.*

Thus:

.3	=	$\frac{3}{10}$	= 3 tenths.
.03	=	$\frac{3}{100}$	= 3 hundredths.
.003	=	$\frac{3}{1000}$	= 3 thousandths.
.0003	=	$\frac{3}{10000}$	= 3 ten-thousandths.
.00003	=	$\frac{3}{100000}$	= 3 hundred-thousandths.
.000003	=	$\frac{3}{1000000}$	= 3 millionths.

We see in the above that the *number of decimal places in a decimal equals the number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. The *decimal point merely separates the fraction on the right from the whole number on the left.*

When a whole number and decimal are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9
hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.

The figures to the *left* of the decimal point represent *whole numbers;* those to the *right* are *decimals.*

In both the decimals and whole numbers, the *units* place is made the *starting point* of notation and numeration. Both whole numbers and decimals *decrease* on the scale of ten to the *right*, and both *increase* on the scale of ten to the *left*. The *first* figure to the *left* of units is *tens*, and the *first* figure to the *right* of units is *tenths*. The *second* figure to the *left* of units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on; the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name, the *decimals* having the ending *ths*, to distinguish them from *whole* numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred-millionths.

The decimals increase to the *left*, on the scale of *ten*, the same as whole numbers; for, if you begin at the 4 in thousandths place in the above table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

135. *Annexing, or taking away, a cipher at the right of a decimal, does not affect its value.*

.5 is $\frac{5}{10}$; .50 is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, .5 = .50.

136. *Inserting a cipher between a decimal and the decimal point, divides the decimal by 10.*

$$.5 = \tfrac{5}{10};\ \tfrac{5}{10} \div 10 = \tfrac{5}{100} = .05.$$

137. *Taking away a cipher from the left of a decimal, multiplies the decimal by 10.*

$$.05 = \tfrac{5}{100};\ \tfrac{5}{100} \times 10 = \tfrac{5}{10} = .5.$$

138. In some cases it is convenient to express a mixed decimal fraction in the form of a common (improper) fraction. To do so it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for, $127.483 = 127\frac{483}{1000} = \frac{127000+483}{1000} = \frac{127483}{1000}$.

ADDITION OF DECIMALS.

139. Addition of decimals is similar in all respects to addition of whole numbers—units are placed under units, tens under tens, etc.; this, of course, brings the decimal points in line, directly under one another. Hence, in placing the numbers to be added, it is only necessary to take care that the *decimal points are in line.* In adding whole numbers, the right-hand figures are always in line; but in adding decimals, the right-hand figures will not be in line unless each decimal contains the same number of figures.

whole numbers		*decimals*		*mixed numbers*	
342		.342		342.032	
4234		.4234		4234.5	
26		.26		26.6782	
3		.03		3.06	
sum 4605	Ans.	*sum* 1.0554	Ans.	*sum* 4606.2702	Ans.

140. Example.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

Solution.—

242.
.36
118.725
1.005
6.
100.1
sum 468.190 Ans.

141. **Rule.**—*Place the numbers to be added so that the decimal points will be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

142. Find the sum of:

(*a*) .2143, .105, 2.3042, and 1.1417.
(*b*) 783.5, 21.473, .2101, and .7816.
(*c*) 21.781, 138.72, 41.8738, .72, and 1.413.
(*d*) .3724, 104.15, 21.417, and 100.042.
(*e*) 200.172, 14.105, 12.1465, .705, and 7.2.
(*f*) 1,427.16, .244, .32, .032, and 10.0041.
(*g*) 2,473.1, 41.65, .7243, 104.067, and 21.073.
(*h*) 4,107.2, .00375, 21.716, 410.072, and .0345.

Ans.

(*a*) 3.7652.
(*b*) 805.9647.
(*c*) 204.5078.
(*d*) 225.9814.
(*e*) 234.3285.
(*f*) 1,437.7601.
(*g*) 2,640.6143.
(*h*) 4,539.02625.

SUBTRACTION OF DECIMALS.

143. As in subtraction of whole numbers, units are placed under units, tens under tens, etc., bringing the decimal points under each other, as in addition of decimals.

Example.—Subtract .132 from .3063.

Solution.—

minuend .3063
subtrahend .132
difference .1743 Ans.

144. Example.—What is the difference between 7.895 and .725?

Solution.—

minuend 7.895
subtrahend .725
difference 7.170 or 7.17. Ans.

145. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	11.000	
subtrahend	.625	
difference	10.375	Ans.

146. Rule.—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them and subtract as before.

EXAMPLES FOR PRACTICE.

147. From:

(*a*)	407.385 take 235.0004.	(*a*)	172.3846.
(*b*)	22.718 take 1.7042.	(*b*)	21.0138.
(*c*)	1,368.17 take 13.6817.	(*c*)	1,354.4883.
(*d*)	70.00017 take 7.000017.	(*d*)	63.000153.
(*e*)	630.630 take .6304.	(*e*)	629.9996.
(*f*)	421.73 take 217.162.	(*f*)	204.568.
(*g*)	1.000014 take .00001.	(*g*)	1.000004.
(*h*)	.783652 take .542314.	(*h*)	.241338.

Ans.

MULTIPLICATION OF DECIMALS.

148. In multiplication of decimals we do not place the decimal points directly under each other as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.825	
multiplier	13	
	2475	
	825	
product	10.725	Ans.

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

149. EXAMPLE.—What is the product of 426 and the decimal .005 ?

SOLUTION.— *multiplicand* 4 2 6
multiplier .0 0 5
product 2.1 3 0 or 2.13. Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

150. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely determine the *number of decimal places*. Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with, and do not change the value.

151. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.— *multiplicand* 1.2 0 5
multiplier 1.1 5
6 0 2 5
1 2 0 5
1 2 0 5
product 1.3 8 5 7 5 Ans.

In this example there are 3 decimal places in the multiplicand and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

152. EXAMPLE.—Multiply .232 by .001.

SOLUTION.— *multiplicand* .2 3 2
multiplier .0 0 1
product .0 0 0 2 3 2 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which gives 232 for the product; but since there are 3 decimal places each in the multiplier and multiplicand, we must prefix 3 ciphers to the 232 to make 3 + 3, or 6, decimal places in the product.

153. **Rule.**—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply*

as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.

EXAMPLES FOR PRACTICE.

154. Find the product of:

(*a*)	.000492 × 4.1418.	(*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	(*d*)	.015477.
(*e*)	178,352 × .01.	(*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

Ans.

DIVISION OF DECIMALS.

155. In division of decimals we pay no attention to the decimal point until after the division has been performed. The *number of decimal places in the dividend must equal (or be made to equal by annexing ciphers) the number of decimal places in the divisor.* Divide exactly as in whole numbers. *Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.*

EXAMPLE.—Divide .625 by 25.

SOLUTION.—

divisor dividend quotient

25) .625 (.025 Ans.

50

125

125

remainder 0

In this example there are no decimal places in the divisor, and three decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25 to make the three decimal places.

156. EXAMPLE.—Divide 6.035 by .05.

divisor dividend quotient

SOLUTION.—

```
.0 5 ) 6.0 3 5 ( 1 2 0.7  Ans.
       5
       ---
       1 0
       1 0
       -----
         3 5
         3 5
         ---
remainder  0
```

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, one decimal place is pointed off in the quotient.

157. EXAMPLE.—Divide .125 by .005.

divisor dividend quotient

SOLUTION.—

```
.0 0 5 ) .1 2 5 ( 2 5  Ans.
         1 0
         -----
          2 5
          2 5
          ---
remainder  0
```

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

158. EXAMPLE.—Divide 326 by .25.

divisor dividend quotient

SOLUTION.—

```
.2 5 ) 3 2 6.0 0 ( 1 3 0 4  Ans.
       2 5
       -----
        7 6
        7 5
        -----
         1 0 0
         1 0 0
         -----
remainder    0
```

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

159. EXAMPLE.—Divide .0025 by 1.25.

divisor dividend quotient

SOLUTION.— 1.2 5) .0 0 2 5 0 (.0 0 2 Ans.

2 5 0

remainder 0

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, i. e., as 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, i. e., as 125. It is clearly evident that the dividend, 25, will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making $4+1$, or 5, decimal places. Since there are five decimal places in the dividend and two decimal places in the divisor, we must point off $5-2$, or 3, decimal places in the quotient. In order to point off three decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

160. **Rule.**—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there be a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still be a remainder, terminate the quotient by the plus sign* (+), *which shows that it can be carried further.*

161. EXAMPLE.—What is the quotient of 199 divided by 15?

divisor dividend quotient

SOLUTION.— $15)199(13+\frac{4}{15}$ Ans.

$$\begin{array}{r} 15 \\ \hline 49 \\ 45 \\ \hline \textit{remainder}\ \ 4 \end{array}$$

Or, $15)199.000(13.266+$ Ans.

$$\begin{array}{r} 15 \\ \hline 49 \\ 45 \\ \hline 40 \\ 30 \\ \hline 100 \\ 90 \\ \hline 100 \\ 90 \\ \hline \textit{remainder}\ \ 10 \end{array}$$

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

162. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place further. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667−. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it was desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain five decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are

seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

163. Divide:

(*a*)	101.6688 by 2.36.	Ans. (*a*)	43.08.
(*b*)	187.12264 by 123.107.	(*b*)	1.52.
(*c*)	.08 by .008.	(*c*)	10.
(*d*)	.0003 by 3.75.	(*d*)	.00008.
(*e*)	.0144 by .024.	(*e*)	.6.
(*f*)	.00375 by 1.25.	(*f*)	.003.
(*g*)	.004 by 400.	(*g*)	.00001.
(*h*)	.4 by .008.	(*h*)	50.

REDUCTION OF DECIMALS.

TO REDUCE A FRACTION TO A DECIMAL.

164. EXAMPLE.— $\frac{3}{4}$ equals what decimal?

SOLUTION.—

$$\begin{array}{r} 4\,)\,3.0\,0 \\ \hline .7\,5 \end{array} \quad \text{or } \tfrac{3}{4} = .75. \quad \text{Ans.}$$

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—

$$\begin{array}{l} 8\,)\,7.0\,0\,0\,(\,.8\,7\,5 \\ \underline{6\,4} \\ \quad 6\,0 \\ \quad \underline{5\,6} \\ \qquad 4\,0 \\ \qquad \underline{4\,0} \\ \qquad\quad 0 \end{array} \quad \text{or } \tfrac{7}{8} = .875. \quad \text{Ans.}$$

165. **Rule.**—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

166. Reduce the following common fractions to decimals:

(*a*)	$\frac{15}{32}$.	Ans. (*a*)	.46875.
(*b*)	$\frac{7}{8}$.	(*b*)	.875.
(*c*)	$\frac{21}{32}$.	(*c*)	.65625.
(*d*)	$\frac{51}{64}$.	(*d*)	.796875.
(*e*)	$\frac{4}{25}$.	(*e*)	.16.
(*f*)	$\frac{5}{8}$.	(*f*)	.625.
(*g*)	$\frac{10}{200}$.	(*g*)	.05.
(*h*)	$\frac{4}{1000}$.	(*h*)	.004.

167. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$, or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule shows what decimal part of a foot 9 inches is.

12)9.00(.75 of a foot. Ans.
84
60
60
0

168. Rule.—I. *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one, and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place further, and if the fifth figure is 5 or greater increase the fourth figure by 1, omitting the signs + and −.*

EXAMPLES FOR PRACTICE.

169. Reduce to the decimal part of a foot:

(*a*)	3 in.	Ans.	(*a*) .25 ft.
(*b*)	$4\frac{1}{2}$ in.		(*b*) .375 ft.
(*c*)	5 in.		(*c*) .4167 ft.
(*d*)	$6\frac{5}{8}$ in.		(*d*) .5521 ft.
(*e*)	11 in.		(*e*) .9167 ft.

TO REDUCE A DECIMAL TO A FRACTION.

170. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

171. Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

172. Reduce the following to common fractions:

(*a*) .125.
(*b*) .625.
(*c*) .3125.
(*d*) .04.
(*e*) .06.
(*f*) .75.
(*g*) .15625.
(*h*) .875.

Ans. (*a*) $\frac{1}{8}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{5}{16}$. (*d*) $\frac{1}{25}$. (*e*) $\frac{3}{50}$. (*f*) $\frac{3}{4}$. (*g*) $\frac{5}{32}$. (*h*) $\frac{7}{8}$.

---

173. To express a decimal approximatively as a fraction having a given denominator:

174. EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

175. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

176. Express:

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans. (*a*) $\frac{5}{8}$. (*b*) $\frac{5}{16}$. (*c*) $\frac{5}{32}$. (*d*) $\frac{49}{64}$. (*e*) $\frac{39}{48}$. (*f*) $\frac{89}{96}$.

---

177. The sign for dollars is \$. It is read *dollars.* \$25 is read *25 dollars.*

Since there are 100 cents in a dollar, 1 cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of

a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*.

Thus, \$25.16 is read *twenty-five dollars and sixteen cents;* \$25.168 is read *twenty-five dollars sixteen cents and eight mills.*

SYMBOLS OF AGGREGATION.

178. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8-3}$, or $13 \times (8-3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13 \times (8-3) = 13 \times 5 = 65.$$

$$13 \times \overline{8-3} = 13 \times 5 = 65.$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101.$$

179. In any series of numbers connected by the signs $+$, $-$, $\times$, and $\div$, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

180. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} = ?$

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication follows 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform all the indicated multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1,296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occur, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

181. Find the values of the following expressions:

(*a*)	$(8 + 5 - 1) \div 4$.	Ans.	(*a*) 3.
(*b*)	$5 \times 24 - 32$.		(*b*) 88.
(*c*)	$5 \times 24 \div 15$.		(*c*) 8.
(*d*)	$144 - 5 \times 24$.		(*d*) 24.
(*e*)	$(1,691 - 540 + 559) \div 3 \times 57$.		(*e*) 10.
(*f*)	$2,080 + 120 - 80 \times 4 - 1,670$.		(*f*) 210.
(*g*)	$\overline{(90 + 60 \div 25)} \times 5 - 29$.		(*g*) 1.
(*h*)	$\overline{90 + 60} \div 25 \times 5$.		(*h*) 1.2.

ARITHMETIC.

(CONTINUED.)

PERCENTAGE.

1. **Percentage** is the process of calculating by *hundredths.*

2. The *term* **per cent.** is an abbreviation of the Latin words *per centum,* which mean *by the hundred.* A certain per cent. of a number is the number of hundredths of that number which is indicated by the number of units in the per cent. Thus, 6 per cent. of 125 is $125 \times \frac{6}{100} = 7.5$; 25 per cent. of 80 is $80 \times \frac{25}{100} = 20$; 43 per cent. of 432 pounds is $432 \times \frac{43}{100} = 185.76$ pounds.

3. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read *six per cent.;* $12\frac{1}{2}\%$ is read *twelve and one-half per cent.*, etc.

When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%.....	.01	$\frac{1}{100}$	150%.....	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%.....	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500%.....	5.00	$\frac{500}{100}$ or 5
5%.....	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}\%$.....	.0025	$\frac{1}{100}$ or $\frac{1}{400}$
10%.....	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}\%$.....	.005	$\frac{1}{100}$ or $\frac{1}{200}$
25%.....	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}\%$.....	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%.....	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}\%$.....	.08$\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%.....	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}\%$.....	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%.....	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}\%$.....	.16$\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%.....	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}\%$.....	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

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4. The names of the different elements used in percentage are: the *base*, the *rate per cent.*, the *percentage*, the *amount*, and the *difference.*

5. The **base** is the number on which the per cent. is computed.

6. The **rate** is the number of hundredths of the base to be taken.

7. The **percentage** is the part, or number of *hundredths*, of the base indicated by the rate; or, the percentage is the result obtained by multiplying the base by the rate.

Thus, when it is stated that 7% of \$25 is \$1.75, \$25 is the base, 7% is the rate, and \$1.75 is the percentage.

8. The **amount** is the sum of the base and percentage.

9. The **difference** is the remainder obtained by subtracting the percentage from the base.

Thus, if a man has \$180, and he earns 6% more, he will have altogether $\$180 + \$180 \times .06$, or $\$180 + \$10.80 = \$190.80$. Here \$180 is the base; 6%, the rate; \$10.80, the percentage; and \$190.80, the *amount.*

Again, if an engine of 125 horsepower uses 16% of it in overcoming friction and other resistances, the amount left for obtaining useful work is $125 - 125 \times .16 = 125 - 20 = 105$ horsepower. Here 125 is the base; 16%, the rate; 20, the percentage; and 105, the *difference.*

10. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate expressed decimally.*

EXAMPLE.—Out of a lot of 300 bushels of apples 76% were sold. How many bushels were sold?

SOLUTION.— 76%, the rate, expressed decimally, is .76; the base is 300; hence, the number of bushels sold, or the percentage, is, by the above rule,

$$300 \times .76 = 228 \text{ bushels.} \quad \text{Ans.}$$

Expressing the rule as a

Formula, $percentage = base \times rate.$

11. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then 1%, or $\frac{1}{100}$, of the number, is $12 \div 6$, or 2. Consequently, if $2 = 1\%$, or $\frac{1}{100}$, 100%, or $\frac{100}{100} = 2 \times 100 = 200$. But, since the same result may be arrived at by dividing 12 by .06, for $12 \div .06 = 200$, it follows that.

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate expressed decimally.*

Formula, *base = percentage ÷ rate.*

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If I sold 228 bushels, how many bushels did I buy?

SOLUTION.—Here 228 is the percentage, and 76%, or .76, is the rate; hence, applying the rule,

$228 \div .76 = 300$ bushels. Ans.

12. When the base and percentage are given, to find the rate, the rate may be found, expressed decimally, by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is $200 \times .01 = 2$. Now, if 1% is 2, 12 is evidently as many per cent. as the number of times that 2 is contained in 12, or $12 \div 2 = 6\%$. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since $12 \div 200 = .06 = 6\%$. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate expressed decimally.*

Formula, *rate = percentage ÷ base.*

EXAMPLE.—Bought 300 bushels of apples and sold 228 bushels. What per cent. of the total number of bushels was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule,

rate $= 228 \div 300 = .76 = 76\%$. Ans.

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base, and 25 is the percentage; hence, applying rule,

$25 \div 875 = .02\frac{6}{7} = 2\frac{6}{7}\%$. Ans.

PROOF.— $875 \times .02\frac{6}{7} = 25$.

EXAMPLES FOR PRACTICE.

13. What per cent. of:

(*a*)	360 is 90?	(*a*)	25%.
(*b*)	900 is 360?	(*b*)	40%.
(*c*)	125 is 25?	(*c*)	20%.
(*d*)	150 is 750?	(*d*)	500%.
(*e*)	280 is 112?	(*e*)	40%.
(*f*)	400 is 200?	(*f*)	50%.
(*g*)	47 is 94?	(*g*)	200%.
(*h*)	500 is 250?	(*h*)	50%.

Ans.

14. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate, expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and 6% is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **8**, the amount is $200 + 12 = 212$. But, the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate expressed decimally.*

Formula, *amount* = *base* $\times (1 +$ *rate*$)$.

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and 10% is the rate, and the amount is required. Hence, applying the rule,

$725 \times 1.10 = \$797.50$. Ans.

15. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **9**, the difference $= 200 - 12 = 188$. But, the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate expressed decimally.*

Formula, *difference* = *base* $\times (1 -$ *rate*$)$.

EXAMPLE.—Bought 300 bushels of apples and sold all but 24% of them. How many bushels were sold?

SOLUTION.—Here 300 is the base, 24% is the rate, and it is desired to find the difference. Hence, applying the rule,

$$300 \times (1 - .24) = 228 \text{ bushels. Ans.}$$

16. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number increased by 6% of itself. Then, it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$, or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate expressed decimally.*

Formula, *base* = *amount* $\div$ (1 + *rate*).

EXAMPLE.—The theoretical discharge of a certain pump when running at a piston speed of 100 feet per minute is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount, and 25% is the rate. Applying rule,

$$\text{actual discharge} = 278{,}910 \div 1.25 = 223{,}128 \text{ gallons. Ans.}$$

17. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate expressed decimally.*

Formula, *base* = *difference* $\div$ (1 − *rate*).

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If there were 72 bushels left unsold, how many bushels did I buy?

SOLUTION.—Here 72 is the difference and 76% is the rate. Applying rule,

$$72 \div (1 - .76) = 300 \text{ bushels. Ans.}$$

EXAMPLE.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference, and 30% is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds. Ans.}$$

18. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount, and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **12**,

$$\text{gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \text{ Ans.}$$

19. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself), and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **12**,

$$\text{loss per cent.} = 1.8 \div 3 = .60 = 60\%. \text{ Ans.}$$

20. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and the final value; divide this difference by the initial value.*

EXAMPLE.—If a man buys a house for $1,860, and some time afterwards builds a barn for 25% of the cost of the house, does he gain or lose, and how much per cent., if he sells both house and barn for $2,100?

SOLUTION.—The cost of the barn was $\$1{,}860 \times .25 = \$465$; consequently, the initial value, or total cost, was $\$1{,}860 + \$465 = \$2{,}325$. Since he sold them for $2,100 he lost $\$2{,}325 - \$2{,}100 = \$225$. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss. Ans.}$$

EXAMPLES FOR PRACTICE.

21. Solve the following:

(*a*)	What is $12\frac{1}{2}\%$ of \$900?	Ans. (*a*)	\$112.50.
(*b*)	What is $\frac{4}{5}\%$ of 627?	(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}\%$ of 54?	(*c*)	18.
(*d*)	101 is $68\frac{2}{3}\%$ of what number?	(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}\%$ of what number?	(*e*)	940.8.
(*f*)	What % of 960 is 160?	(*f*)	$16\frac{2}{3}\%$.
(*g*)	What % of \$3,606 is $\$450\frac{3}{4}$?	(*g*)	$12\frac{1}{2}\%$.
(*h*)	What % of 280 is 112?	(*h*)	40%.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved?

Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased $6\frac{1}{2}\%$, how many revolutions per minute would it then make? Ans. 134.19 rev.

3. The list price of a lot of silk goods is \$1,400, of some laces \$1,150, and of some calico \$340. If 25% discount was allowed on the silk, 22% on the laces, and $12\frac{1}{2}\%$ on the calico, what was the actual cost of the purchase? Ans. \$2,244.50.

4. If I loan a man \$1,100, and this is $18\frac{1}{2}\%$ of the amount that I have on interest, how much money have I on interest? Ans. \$5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H. P.

6. By adding a condenser to a steam engine, the power was increased 14% and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower, and required $3\frac{1}{2}$ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour, with the condenser, assuming the engine to run full power? Ans. 159.6 pounds.

DENOMINATE NUMBERS.

22. A **denominate number** is a concrete number, and may be either simple or compound; as, 8 quarts; 5 feet; 10 inches, etc.

23. A **simple denominate number** consists of units of but one denomination; as, 16 cents; 10 hours; 5 dollars, etc.

24. A **compound denominate number** consists of units of two or more denominations of a similar kind; as, 3 yards, 2 feet, 1 inch; 34 square feet, 57 square inches.

25. In **whole numbers** and in **decimals** the law of increase and decrease is on the scale of 10, but in **compound** or **denominate numbers** the scale varies.

26. A **measure** is a *standard unit*, established by *law* or *custom*, by which quantity of any kind is measured. The standard unit of **dry measure** is the Winchester bushel; of **weight,** the pound; of **liquid measure,** the gallon, etc.

27. Measures are of six kinds:

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

28. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE.

	Abbreviation.									
12 inches (in.)	= 1 foot . .	ft.	in.		ft.		yd.		rd.	fur. mi.
3 feet . . .	= 1 yard .	yd.	36	=	3	=	1			
5.5 yards . .	= 1 rod . .	rd.	198	=	$16\frac{1}{2}$	=	5.5	=	1	
40 rods . . .	= 1 furlong	fur.	7,920	=	660	=	220	=	40	= 1
8 furlongs .	= 1 mile .	mi.	63,360	=	5,280	=	1,760	=	320	= 8 = 1

SURVEYOR'S LINEAR MEASURE.

TABLE.

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods 100 links	= 1 chain	ch.
80 chains	= 1 mile	mi.

mi.		ch.		rd.		li.		in.
1	=	80	=	320	=	8,000	=	63,360

29. The linear unit, generally used by surveyors, is **Gunter's chain,** which is equal to 4 rods, or 66 feet.

30. An **engineer's chain,** used by civil engineers, is 100 feet long, and consists of 100 links. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE.

144 square inches (sq. in.) . . . = 1 square foot . . . . sq. ft.
9 square feet . . . . . = 1 square yard . . . . sq. yd.
30¼ square yards . . . . . = 1 square rod . . . . sq. rd.
160 square rods . . . . . = 1 acre . . . . . A.
640 acres . . . . . = 1 square mile . . . . sq. mi.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

SURVEYOR'S SQUARE MEASURE.

TABLE.

625 square links (sq. li.) . . . . = 1 square rod . . . . sq. rd.
16 square rods . . . . . = 1 square chain . . . sq. ch.
10 square chains . . . . . = 1 acre . . . . . A.
640 acres. . . . . = 1 square mile . . . . sq. mi.
36 square miles (6 mi. square) . . = 1 township . . . . . Tp.

sq. mi.	A.	sq. ch.	sq. rd.	sq. li.
1	= 640	= 6,400	= 102,400	= 64,000,000

CUBIC MEASURE.

TABLE.

1,728 cubic inches (cu. in.) . . . = 1 cubic foot . . . . cu. ft.
27 cubic feet . . . . . = 1 cubic yard . . . . cu. yd.
128 cubic feet . . . . . = 1 cord . . . . . cd.
24¾ cubic feet . . . . . = 1 perch . . . . . P.

cu. yd.	cu. ft.	cu. in.
1	= 27	= 46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE.

16 ounces (oz.) . . . . . = 1 pound . . . . . lb.
100 pounds . . . . . = 1 hundredweight . . . cwt.
20 cwt., or 2,000 lb. . . . . = 1 ton . . . . . T.

T.	cwt.	lb.	oz.
1	= 20	= 2,000	= 32,000

31. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles. One avoirdupois pound contains 7,000 grains.

LONG TON TABLE.

16 ounces	=	1 pound	lb.
112 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,240 lb.	=	1 ton	T.

32. In all the calculations throughout this and the following sections, 2,000 pounds will be considered 1 ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE.

24 grains (gr.)	=	1 pennyweight	pwt.
20 pennyweights	=	1 ounce	oz.
12 ounces	=	1 pound	lb.

lb.	oz.	pwt.	gr.
1	= 12 =	240 =	5,760

33. Troy weight is used in weighing gold and silverware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE.

4 gills (gi.)	=	1 pint	pt.
2 pints	=	1 quart	qt.
4 quarts	=	1 gallon	gal.
31½ gallons	=	1 barrel	bbl.
2 barrels, or 63 gallons	=	1 hogshead	hhd.

hhd.	bbl.	gal.	qt.	pt.	gi.
1 =	2 =	63 =	252 =	504 =	2,016

DRY MEASURE.

TABLE.

2 pints (pt.)	=	1 quart	qt.
8 quarts	=	1 peck	pk.
4 pecks	=	1 bushel	bu.

bu.	pk.	qt.	pt.
1 =	4 =	32 =	64

MEASURE OF TIME.

TABLE.

60 seconds (sec.) . . . . . = 1 minute . . . . . min.
60 minutes . . . . . = 1 hour . . . . . hr.
24 hours . . . . . = 1 day . . . . . da.
7 days . . . . . = 1 week . . . . . wk.
365 days / 12 months . . . . . = 1 common year . . . . yr.
366 days . . . . . = 1 leap year.
100 years . . . . . = 1 century.

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE.

60 seconds (″) . . . . . = 1 minute . . . . . ′
60 minutes . . . . . = 1 degree . . . . . °
90 degrees . . . . . = 1 right angle or quadrant ∟
360 degrees . . . . . = 1 circle . . . . . cir.

1 cir. = 360° = 21,600′ = 1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE.

10 mills (m.) . . . . . = 1 cent . . . . . ct.
10 cents . . . . . = 1 dime . . . . . d.
10 dimes . . . . . = 1 dollar . . . . . $.
10 dollars . . . . . = 1 eagle . . . . . E.

E.	$	d.	ct.	m.
1	= 10	= 100	= 1,000	= 10,000

MISCELLANEOUS TABLE.

12 things are 1 dozen.
12 dozen are 1 gross.
12 gross are 1 great gross.
2 things are 1 pair.
20 things are 1 score.
1 league is 3 miles.
1 fathom is 6 feet.
1 meter is nearly 39.37 inches.
1 hand is 4 inches.
1 palm is 3 inches.
1 span is 9 inches.
24 sheets are 1 quire.
20 quires, or 480 sheets, are 1 ream.
1 bushel contains 2,150.4 cubic in.
1 U. S. standard gallon (also called a wine gallon) contains 231 cubic in.
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all the above tables.

REDUCTION OF DENOMINATE NUMBERS.

34. **Reduction** of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination, or from a lower to a higher—either is reduction. As

2 hours = 120 minutes.
32 ounces = 2 pounds.

35. **Principle.**—Denominate numbers are changed to lower denominations by *multiplying*, and to higher denominations by *dividing*.

To reduce denominate numbers to lower denominations:

36. EXAMPLE.—Reduce 5 yd. 2 ft. 7 in. to inches.

SOLUTION.—

yd.	ft.	in.
5	2	7
3		
15 ft.		
2 ft.		
17 ft.		
12		
34		
17		
204 in.		
7 in.		
211 inches. Ans.		

EXPLANATION.—Since there are 3 feet in 1 yard, in 5 yards there are 5×3 or 15 feet, and 15 feet plus 2 feet = 17 feet. There are 12 inches in a foot; therefore, $12 \times 17 = 204$ inches, and 204 inches plus 7 inches = 211 inches = number of inches in 5 yards 2 feet and 7 inches.

37. EXAMPLE.—Reduce 6 hours to seconds.

SOLUTION.—

6	hours.
60	
360	minutes.
60	
21600	seconds. Ans.

EXPLANATION.—As there are 60 minutes in 1 hour, in 6 hours there are 6×60, or 360, minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

38. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet, as

rd.	yd.	ft.	in.
3	0	0	6

39. **Rule.**—*Multiply the number representing the highest denomination by the number of units in the next lower required to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

40. Reduce:

(*a*)	4 rd. 2 yd. 2 ft. to ft.	Ans.	(*a*) 74 ft.
(*b*)	4 bu. 3 pk. 2 qt. to qt.		(*b*) 154 qt.
(*c*)	13 rd. 5 yd. 2 ft. to ft.		(*c*) 231.5 ft.
(*d*)	5 mi. 100 rd. 10 ft. to ft.		(*d*) 28,060 ft.
(*e*)	8 lb. 4 oz. 6 pwt. to gr.		(*e*) 48,144 gr.
(*f*)	52 hhd. 24 gal. 1 pt. to pt.		(*f*) 26,401 pt.
(*g*)	5 cir. 16° 20′ to minutes.		(*g*) 108,980′.
(*h*)	14 bu. to qt.		(*h*) 448 qt.

To reduce lower to higher denominations:

41. EXAMPLE.—Reduce 211 inches to higher denominations.

SOLUTION.—

$$\begin{array}{r} 12\,)\,2\,1\,1 \text{ in.} \\ \hline 3\,)\,1\,7 \text{ ft.} + 7 \text{ in.} \\ \hline 5 \text{ yd.} + 2 \text{ ft.} \end{array}$$ Ans.

EXPLANATION.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There are 3 feet in 1 yard; therefore, 17 feet divided by 3 = 5 yards

and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

42. Example.—Reduce 15,735 grains Troy weight to higher denominations.

Solution.—

24) 15735 gr. (655 pwt.
144
133
120
135
120
15 gr.

20) 655 pwt. (32 oz.
60
55
40
15 pwt.

12) 32 oz. (2 lb.
24
8 oz.

Explanation.—There are 24 grains in 1 pennyweight, and in 15,735 grains there are as many pennyweights as 24 is contained in 15,735, or 655 pennyweights and 15 grains remaining. There are 20 pennyweights in 1 ounce, and in 655 pennyweights there are 32 ounces and 15 pennyweights remaining. There are 12 ounces in 1 pound, and in 32 ounces there are 2 pounds and 8 ounces remaining. The last quotient and the three remainders constitute the answer, 2 pounds 8 ounces 15 pennyweights 15 grains.

The above problem is worked out by long division, because the numbers are too large to solve easily by short division. The student may use either method.

43. **Rule.**—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by the number of units of its denomination required to make one unit of the next higher. Continue until the highest*

denomination is reached, or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.

EXAMPLES FOR PRACTICE.

44. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 7,896 cu. ft. to cd.; (*e*) 17,651″; (*f*) 1,120 cu. ft. to cd.; (*g*) 8,000 gi.; (*h*) 36,450 lb.

Ans.
- (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
- (*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
- (*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
- (*d*) 61 cd. 88 cu. ft.
- (*e*) 4° 54′ 11″.
- (*f*) 8 cd. 96 cu. ft.
- (*g*) 3 hhd. 61 gal.
- (*h*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

45. Example.—Find the sum of 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

Solution.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

Explanation.—Begin to add at the right-hand column: $4+13+13+12 = 42$ ounces; as 16 ounces make 1 pound, 42 ounces $\div 16 = 2$ and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under ounce column and add 2 pounds to the next or pound column. Then, $2+27+50+12+46 = 137$ pounds; as 100 pounds make a hundredweight, $137 \div 100 = 1$ hundredweight and a remainder of 37 pounds. Place the 37 under the pounds column, and add 1 hundredweight to the next or hundredweight column. Next, $1+12+8+3 = 24$ hundredweight.

20 hundredweight make a ton; therefore $24 \div 20 = 1$ ton and 4 hundredweight remaining. Hence, the sum is 1 ton 4 hundredweight 37 pounds 10 ounces. Ans.

46. EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

SOLUTION.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	$3\frac{1}{2}$	0	2	
or	26	3	1	8	Ans.

EXPLANATION.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the next column plus 2 yards = 9 yards, or $9 \div 5\frac{1}{2} = 1$ rod and $3\frac{1}{2}$ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the $\frac{1}{2}$ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches. Ans.

47. EXAMPLE.—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

SOLUTION.—When 47 ft. is reduced it equals 2 rd. 4 yd. 2 ft. which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	$1\frac{1}{2}$	1	10	
or	6	2	0	4	Ans.

48. **Rule.**—*Place the numbers so that like denominations are under each other. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added, and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

49. What is the sum of:

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{2}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{5}{8}$ lb.?

(*d*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 T. 11 cwt. 49 lb. 14 oz.; 16 T. 47 lb. 13 oz.; 20 T. 13 cwt. 14 lb. 6 oz.; 11 T. 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans. (*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 T. 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

50. Example.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

Solution.—

rd.	yd.	ft.	in.	
21	2	2	$6\frac{1}{2}$	
9	4	0	$10\frac{1}{4}$	
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$	Ans.

Explanation.—Since $10\frac{1}{4}$ inches cannot be taken from $6\frac{1}{2}$ inches, we must borrow 1 foot or 12 inches from the 2 feet in the next column and add it to the $6\frac{1}{2}$. $6\frac{1}{2}+12 = 18\frac{1}{2}$. $18\frac{1}{2}$ inches $-10\frac{1}{4}$ inches $= 8\frac{1}{4}$ inches. Then, 0 from the 1 remaining foot $=$ 1 foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or $5\frac{1}{2}$ yards, from 21 rods and add it to 2. $2+5\frac{1}{2} = 7\frac{1}{2}$; $7\frac{1}{2}-4 = 3\frac{1}{2}$ yards. 9 rods from 20 rods $=$ 11 rods. Hence, the remainder is 11 rods $3\frac{1}{2}$ yards 1 foot $8\frac{1}{4}$ inches. Ans.

To avoid fractions as much as possible, we reduce the $\frac{1}{2}$ yard to inches, obtaining 18 inches; this added to $8\frac{1}{4}$ inches gives $26\frac{1}{4}$ inches, which equals 2 feet $2\frac{1}{4}$ inches. Then, 2 feet $+1$ foot $=$ 3 feet $=$ 1 yard, and 3 yards $+1$ yard $=$ 4

yards. Hence, the above answer becomes 11 rods 4 yards 0 feet $2\frac{1}{4}$ inches.

51. EXAMPLE.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft. ?

SOLUTION.— 47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	$3\frac{1}{2}$	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

52. EXAMPLE.—How many years, months, days, and hours between 4 o'clock P. M. of June 16, 1868, and 10 o'clock A. M., September 29, 1891?

SOLUTION.—

yr.	mo.	da.	hr.	
1891	8	28	10	
1868	5	15	16	
23	3	12	18	Ans.

EXPLANATION.—Counting 24 hours in 1 day, 4 o'clock P. M. is the 16th hour from the beginning of the day, or midnight. On September 29, 8 months and 28 days have elapsed, and on June 16, 5 months and 15 days. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

53. **Rule.**—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be borrowed from the minuend of the next higher denomination, reduced, and added to it.*

EXAMPLES FOR PRACTICE.

54. From:

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.

(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.

(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.

(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.

(*e*) 100 bu. take 28 bu. 2 pk 5 qt. 1 pt.

(*f*) 14 mi. 34 rd. 16 yd. 13 ft. 11 in. take 3 mi. 27 rd. 11 yd. 4 ft. 10 in.

Ans. { (*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

55. EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

SOLUTION.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

EXPLANATION.— 15 grains $\times 12 = 180$ grains. $180 \div 24 = 7$ pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next. Now, $13 \times 12 + 7 = 163$ pennyweights; $163 \div 20 = 8$ ounces and 3 pennyweights remaining. Then, $5 \times 12 + 8 = 68$ ounces; $68 \div 12 = 5$ pounds and 8 ounces remaining. Then, $7 \times 12 + 5 = 89$ pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains. Ans.

56. **Rule.**—*Multiply the number representing each denomination by the multiplier and reduce each product to the next higher denomination, writing the remainders under each denomination, and carry the quotient to the next, as in Addition of Denominate Numbers.*

57. In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the example of Art. **55**, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. $43{,}047 \times 1.2 = 51{,}656.4$ gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, $43{,}047 \times 12 = 516{,}564$ gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as above. Either method may be used.

EXAMPLES FOR PRACTICE.

58. Multiply:

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 4 wk. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans. (*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 113 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

59. EXAMPLE.—Divide 48 lb. 11 oz. 6 pwt. by 8.

SOLUTION.—

	lb.	oz.	pwt.	gr.
8)	48	11	6	0
	6 lb.	1 oz.	8 pwt.	6 gr. Ans.

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 ounces once, with 3 ounces remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pennyweights; 66 pennyweights $\div 8 = 8$ pennyweights and 2 remaining; 2×24 grains $= 48$ grains; 48 grains $\div 8 = 6$ grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains. Ans.

EXAMPLE.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

SOLUTION.—

	lb.	oz.	pwt.
6)	2	8	10
		5 oz.	8 pwt. 8 gr. Ans.

EXPLANATION.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds $= 24$ ounces, and 24 ounces $+ 8$ ounces $= 32$ ounces; 32 ounces $\div 6 = 5$ ounces and 2 ounces over. 2 ounces $= 40$ pennyweights; 40 pennyweights $+ 10$ pennyweights $= 50$ pennyweights, and 50 pennyweights $\div 6 = 8$ pennyweights and 2 pennyweights over. 2 pennyweights $= 48$ grains, and 48 grains $\div 6 = 8$ grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains. Ans.

60. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

SOLUTION.—

rd. yd. ft. rd. yd. ft. in.

112)820 4 2(7 1 2 5.143 Ans.

784

36 rd. rem.

5.5

180

180

198.0 yd.

4

112)202 yd. (1 yd.

112

90 yd. rem.

3

270 ft.

2 ft.

112)272 ft. (2 ft.

224

48 ft. rem.

12

96

48

112)576 in. (5.1428+ in. or 5.143 in.

560

160

112

480

448

320

224

960

896

64

EXPLANATION.—The first quotient is 7 rods with 36 rods remaining. $5.5 \times 36 = 198$ yards; 198 yards + 4 yards = 202 yards; 202 yards $\div 112 = 1$ yard and 90 yards remaining. $90 \times 3 = 270$ feet; 270 feet + 2 feet = 272 feet; 272 feet $\div 112 = 2$ feet, and 48 feet remaining; $48 \times 12 = 576$ inches; 576 inches $\div 112 = 5.143$ inches, nearly. Ans.

The preceding example is solved by long division, because

the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

61. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

62. Divide:

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 T. 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans. (*a*) 17 mi. $41\frac{7}{11}$ rd.
(*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
(*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
(*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in.
(*e*) 12 mi. 112 rd. 2 yd.
(*f*) 6 T. 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
(*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
(*h*) 1 mi. $38\frac{28}{25}$ rd.

INVOLUTION.

63. Involution is the process of multiplying a number by itself one or more times. The product obtained by multiplying a number by itself is called a **power** of that number.

Thus, the *second* power of 3 is 9, since 3×3 are 9.

The *third* power of 3 is 27, since $3\times3\times3$ are 27.
The *fifth* power of 2 is 32, since $2\times2\times2\times2\times2$ are 32.

64. An **exponent** is a *small figure* placed to the *right* and a little *above* a number to show to what *power* it is to be raised, or how many times the number is to be used as a factor, as the small figures ^{2,} ^{3,} and ^{5} below.

Thus, $3^2 = 3\times3 = 9.$
$3^3 = 3\times3\times3 = 27.$
$2^5 = 2\times2\times2\times2\times2 = 32.$

65. The **root** of a number is that number which, used the required number of times as a factor, produces the number. In the above cases 3 is a root of 9, since 3×3 are 9. It is also a root of 27, since $3\times3\times3$ are 27. Also, 2 is a root of 32, since $2\times2\times2\times2\times2$ are 32.

66. The *second* power of a number is called its **square.**
Thus, 5^2 is called the *square* of 5, or 5 *squared*, and its value is $5\times5 = 25.$

67. The *third* power of a number is called its **cube.**
Thus, 5^3 is called the *cube* of 5, or 5 *cubed*, and its value is $5\times5\times5 = 125.$

To find any power of a number:

68. EXAMPLE.—What is the third power, or cube, of 35?

SOLUTION.—

$35\times35\times35,$
or
$$\begin{array}{r} 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \\ 35 \\ \hline 6125 \\ 3675 \\ \hline \end{array}$$
cube = 42875 Ans.

EXAMPLE.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15$,

or

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \\ 15 \\ \hline 16875 \\ 3375 \\ \hline 50625 \end{array}$$

fourth power = 50625 Ans.

69. EXAMPLE.— 1.2^3 = what?

SOLUTION.— $1.2 \times 1.2 \times 1.2$,

or

$$\begin{array}{r} 1.2 \\ 1.2 \\ \hline 1.44 \\ 1.2 \\ \hline 288 \\ 144 \\ \hline 1.728 \end{array}$$

cube = 1.728 Ans.

70. EXAMPLE.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $(\frac{3}{8})^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

71. **Rule.—I.** *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

72. Raise the following to the powers indicated:

		Ans.	
(*a*)	85^2.	(*a*)	7,225.
(*b*)	$(1\frac{2}{13})^2$.	(*b*)	$1\frac{44}{169}$.
(*c*)	6.5^2.	(*c*)	42.25.
(*d*)	14^4.	(*d*)	38,416.
(*e*)	$(\frac{3}{4})^3$.	(*e*)	$\frac{27}{64}$.
(*f*)	$(\frac{5}{6})^3$.	(*f*)	$1\frac{25}{216}$.
(*g*)	$(\frac{7}{2})^3$.	(*g*)	$3\frac{43}{8}$.
(*h*)	1.4^5.	(*h*)	5.37824.

EVOLUTION.

73. **Evolution** is the reverse of involution. It is the process of finding the root of a number which is considered as a power.

74. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2, or $2^2 = 4$.

75. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or $3^3 = 27$.

76. The **radical sign** $\sqrt{\ }$, when placed before a number, indicates that some root of that number is to be found.

77. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.
$\sqrt[3]{125}$ denotes the *cube root* of 125.
$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

78. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

SQUARE ROOT.

79. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the largest number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root.

This is done by pointing off the number into *periods* of two figures each, *beginning at the right*. The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83′74′08′01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures, since there are (5′06′25) 3 periods.

80. Example.—Find the square root of 31,505,769.

Solution.—

				root	
				3 1′5 0′5 7′6 9 (5 6 1 3	Ans.
(*a*)	5				
	5	(*b*)	2 5		
(*d*)	1 0 0	(*c*)	6 5 0		
	6		6 3 6		
	1 0 6	(*e*)	1 4 5 7		
	6		1 1 2 1		
	1 1 2 0		3 3 6 6 9		
	1		3 3 6 6 9		
	1 1 2 1		0		
	1				
	1 1 2 2 0				
	3				
	1 1 2 2 3				

Explanation.—Pointing off into periods of two figures each, it is seen that there are four figures in the root. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract, and obtain 6 as a remainder.

Bring down the next period, 50, and annex it to the remainder, 6, as shown at (*c*), which we call the **dividend.** Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, which we call the **trial divisor.** Divide the dividend (*c*) by the trial divisor (*d*) and obtain 6, which is probably the next figure of the root. Write 6 in the root, as shown, and also add it

to 100, the trial divisor, making it 106. This is called the **complete divisor.**

Multiply this by 6, the second figure in the root, and subtract the result from the dividend (*c*). The remainder is 14, to which annex the next period, making it 1,457, as shown at (*e*), which we call the **new dividend.** Add the second figure of the root to the complete divisor, 106, and annex a cipher, thus getting 1,120. Dividing 1,457 by 1,120, we get 1 as the next figure of the root. Adding this last figure of the root to 1,120, multiplying the result by it, and subtracting from 1,457, the remainder is 336.

Annexing the next and last period, 69, the result is 33,669. Now, adding the last figure of the root to 1,121, and annexing a cipher as before, the result is 11,220. Dividing 33,669 by 11,220, the result is 3, the fourth figure in the root. Adding it to 11,220 and multiplying the sum by it, the result is 33,669. Subtracting, there is no remainder; hence, $\sqrt{31,505,769} = 5,613$.

81. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$, $.13^2 = .0169$, $.751^2 = .564001$, etc.

82. It will also be noticed that the number squared is always less than the decimal. Hence, if it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to determine the number of figures in the root of a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

83. EXAMPLE.—What is the square root of .000576?

SOLUTION.—

		root
	2	.0 0′0 5′7 6 (.0 2 4 Ans.
	2	4
	40	176
	4	176
	44	0

EXPLANATION.—Beginning at the decimal point, and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

84. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

85. EXAMPLE.—What is the square root of 3? Find the result to five decimal places.

SOLUTION.—

		root
1	3.0 0'0 0'0 0'0 0'0 0	(1.7 3 2 0 5 + Ans.
1	1	
20	200	
7	189	
27	1100	
7	1029	
340	7100	
3	6924	
343	176 0000	
3	173 2025	
3460	2 7975	
2		
3462		
2		
346400		
5		
346405		

EXPLANATION.—Annexing five periods of two ciphers each to the right of the decimal point, the first figure of the root is 1. To get the second figure we find that, in dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20, and multiply 29 by 9; the result is 261, a result which is considerably larger than 200; hence, 9 is too large. In the same way it is found that 8 is also too large. Trying 7, 7 times 27 are 189, a result smaller than

200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor, 34,640, is greater than the new dividend, 17,600. In a case of this kind we annex another cipher to 34,640, thereby making it 346,400, and bring down the next period, making the 17,600, 1,760,000. The next figure of the root is 5, and, as we now have five decimal places, we will stop.

The square root of 3 to five decimal places is, then, 1.73205 +.

86. Example.—What is the square root of .3 to five decimal places?

Solution.—

		root
5	.3 0'0 0'0 0'0 0'0 0	(.5 4 7 7 2 + Ans.
5	2 5	
100	500	
4	416	
104	8400	
4	7609	
1080	79100	
7	76629	
1087	247100	
7	219084	
10940	28016	
7		
10947		
7		
109540		
2		
109542		

Explanation.—In the above example we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

87. If it is required to find the square root of a mixed number, begin at the decimal point, and point off the periods both ways. The manner of finding the root will then be exactly the same as in the previous cases.

88. Example.—What is the square root of 258.2449?

Solution.—

```
                                  root
1        2'5 8.2 4'4 9 ( 1 6.0 7   Ans.
1        1
20       158
 6       156
26          22449
 6          22449
3200             0
   7
3207
```

Explanation.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root, and annex a cipher to 320, making it 3,200. Then, bringing down the next period, 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07.

89. Proof.—*To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.*

90. Rule.—I. *Begin at units place, and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there be any.*

II. *Find the greatest number whose square is contained in the first, or left-hand, period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period; then, annex the second period to the remainder.

III. *Add the first figure of the root to the number in the first column on the left, and annex a cipher to the result; this is the trial divisor. Divide the dividend by the trial divisor for the second figure in the root, and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root and*

subtract this result from the dividend. (*If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.*) *Now bring down the third period, and annex it to the last remainder for a new dividend. Add the second figure of the root to the complete divisor, and annex a cipher for a new trial divisor.*

IV. *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods, and continue the operation.*

V. *If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

VI. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

91. **Short Method.**—If the number whose root is to be extracted is not an exact square, the root will be an interminable decimal. It is then usual to extract the root to a certain number of decimal places. In such cases, the work may be greatly shortened as follows: Determine to how many decimal places the work is to be carried, say 5, for example; add to this the number of places in the integral part of the root, say 2, for example, thus determining the number of figures in the root, in this case $5+2=7$. Divide this number by 2 and take the next higher number. In the above case, we have $7\div 2=3\frac{1}{2}$; hence, we take 4, the next higher number. Now extract the root in the usual manner until the same number of figures has been obtained as was expressed by the number obtained above, in this case 4. Then form the trial divisor in the usual manner, but omitting to add the cipher; divide the last remainder by the trial divisor as in long division, obtaining as many figures of the quotient as there are remaining figures of the root, in this case $7-4=3$. The remainder so obtained is the remaining figures of the root.

Consider the example in Art. **86.** Here there are 5 figures in the root. We therefore extract the root to 3 places in the usual manner, obtaining .547 for the first three root figures. The next trial divisor is 1,094 (with the cipher omitted) and the last remainder is 791. Then, $791 \div 1{,}094 = .723$, and the next two figures of the root are 72, the whole root being $.54772+$. Always carry the division one place further than desired, and if the last figure is 5 or greater increase the preceding figure by 1. This method should not be used unless the root contains five or more figures.

If the last figure of the root found in the regular manner is a cipher, carry the process one place further before dividing as described above.

EXAMPLES FOR PRACTICE.

92. Find the square root of:

(*a*)	186,624.	(*a*)	432.
(*b*)	2,050,624.	(*b*)	1,432.
(*c*)	29,855,296.	(*c*)	5,464.
(*d*)	.0116964.	(*d*)	.1081+.
(*e*)	198.1369.	(*e*)	14.0761.
(*f*)	994,009.	(*f*)	997.
(*g*)	2.375 to four decimal places.	(*g*)	1.5411.
(*h*)	1.625 to three decimal places.	(*h*)	1.275.
(*i*)	.3025.	(*i*)	.55.
(*j*)	.571428.	(*j*)	.7559+.
(*k*)	.78125.	(*k*)	.8839.

Ans.

CUBE ROOT.

93. In the same manner as in the case of square root, it can be shown that the periods into which a number is divided, whose cube root is to be extracted, must contain three figures, except that the first, or left-hand, period of a whole or mixed number may contain one, two, or three figures.

94. Example.—What is the cube root of 375,741,853,696?

Solution.—

(1)	(2)	(3) root
7	49	375'741'853'696(7216 Ans
7	98	343
14	14700	32741
7	424	30248
210	15124	2493853
2	428	1557361
212	1555200	936492696
2	2161	936492696
214	1557361	0
2	2162	
2160	155952300	
1	129816	
2161	156082116	
1		
2162		
1		
21630		
6		
21636		

Explanation.—Write the work in three columns, as follows: On the right, place the number whose cube root is to be extracted, and point it off into periods of three figures each. Call this column (3). Find the largest number whose cube is less than or equal to the first period, in this case 7. Write the 7 on the right as shown, for the first figure of the root, and also on the extreme left at the head of column (1). Multiply the 7 in column (1) by the first figure of the root, 7, and write the product, 49, at the head of column (2). Multiply the number in column (2) by the first figure of the root, 7, and write the product, 343, under the figures in the first period. Subtract and bring down the next period, obtaining 32,741 for the dividend. Add the first figure of the root to the number in column (1), obtaining 14, which call the *first correction*. Multiply the first correction by the first figure of the root, add the product to the number in column (2), and obtain 147. Add the first figure of the root to the first correction, and obtain 21, which call

the *second correction.* Annex *two* ciphers to the number in column (2), and obtain 14,700 for the trial divisor; also annex *one* cipher to the second correction, and obtain 210. Dividing the dividend by the trial divisor, we obtain $\frac{32,741}{14,700} = 2+$, and write the 2 as the second figure of the root. Add the 2 to the second correction, and obtain 212, which multiplied by the second figure of the root and added to the trial divisor, gives 15,124, the complete divisor. This last result multiplied by the second figure of the root and subtracted from the dividend, gives a remainder of 2,493. Annexing the third period, we obtain 2,493,853 for the new dividend. Adding the second figure of the root to the number in column (1) we get 214 as the new first correction; this multiplied by the second figure of the root and added to the complete divisor, gives 15,552. Adding the second figure of the root to the first new correction gives 216 as the second new correction. Annexing two ciphers to the number in column (2) gives 1,555,200, the new trial divisor. Annexing one cipher to the second new correction gives 2,160. Dividing the new dividend by the new trial divisor we obtain $\frac{2,493,853}{1,555,200} = 1+$, and write 1 as the third figure of the root. The remainder of the work should be perfectly clear from what has preceded.

95. In extracting the cube root of a decimal, proceed as above, taking care that each period contains *three* figures. Begin the pointing off at the decimal point, going towards the right. If the last period does not contain three figures, annex ciphers until it does.

96. Example.—What is the cube root of .009129329?

Solution.—

			root
2	4	.009'129'329	(.209
2	8	8	
4	120000	1129329	
2	5481	1129329	
600	125481	0	
9			
609			

EXPLANATION.—Beginning at the decimal, and pointing off as shown, the largest number whose cube is less than 9 is seen to be 2; hence, 2 is the first figure of the root. When finding the second figure, it is seen that the trial divisor, 1,200, is greater than the dividend; hence, write a cipher for the second figure of the root; bring down the next period to form the new dividend; annex two ciphers to the trial divisor to form a new trial divisor; also, annex one cipher to the 60 in the first column. Dividing the new dividend by the new trial divisor, we get $\frac{1129329}{120000} = 9+$, and write 9 as the third figure of the root. Complete the work as before.

97. EXAMPLE.—What is the cube root of 78,347.809639?

SOLUTION.—

		root
4	16	78′347.809′639(42.79
4	32	64
8	4800	14347
4	244	10088
120	5044	4259809
2	248	3766483
122	529200	493326639
2	8869	493326639
124	538069	0
2	8918	
1260	54698700	
7	115371	
1267	54814071	
7		
1274		
7		
12810		
9		
12819		

EXPLANATION.—Since the above is a mixed number, begin at the decimal point and point off periods of three figures

each, in both directions. The first period contains but two figures, and the largest number whose cube is less than 78 is 4; consequently, 4 is the first figure of the root. The remainder of the work should be perfectly clear. When dividing the dividend by the trial divisor for the third figure of the root, the quotient was 8+, but, on trying it, it was found that 8 was too large, the complete divisor being considerably larger than the trial divisor. Therefore, 7 was used instead of 8.

98. EXAMPLE.—What is the cube root of 5 to five decimal places?

SOLUTION.—

			root
1	1	5.000'000'000'000'000	(1.70997+
1	2	1	
2	300	4000	
1	259	3913	
30	559	87000000	
7	308	78443829	
37	8670000	8556171000	
7	45981	7889992299	
44	8715981	666178701000	
7	46062	614014317973	
5100	876204300	52164383027	
9	461511		
5109	876665811		
9	461592		
5118	877127403 00		
9	3590839		
51270	877163311 39		
9			
51279			
9			
51288			
9			
512970			
7			
512977			

Explanation.—In the above example, we annex five periods of ciphers, of three ciphers each, to the 5 for the decimal part of the root, placing the decimal point between the 5 and the first cipher. Since it is easy to see that the next figure of the root will be 5, we increase the last figure by 1, obtaining 1.70998 for the correct root to 5 decimal places. Ans.

99. Example.—What is the cube root of .5 to four decimal places?

Solution.—

			root
7	49	.500'000'000'000	(.7987+
7	98	343	
14	14700	157000	
7	1971	150039	
210	16671	6961000	
9	2052	5638257	
219	1872300	1322743000	
9	7119	1321748953	
228	1879419	994047	
9	7128		
2370	188654700		
3	166579		
2373	188821279		
3			
2376			
3			
23790			
7			
23797			

Explanation.—In the above example, we annex two ciphers to the .5 to complete the first period, and three periods of three ciphers each. The cube root of 500 is 7; this we write as the first figure of the root. The remainder of the work should be perfectly plain from the explanations of the preceding examples.

100. Example.—What is the cube root of .05 to four decimal places?

Solution.—

			root
3	9	.05'000'000'000	(.3684+
3	18	27	
6	2700	23000	
3	576	19656	
90	3276	3344000	
6	612	3180032	
96	388800	163968000	
6	8704	162685504	
102	397504	1282496	
6	8768		
1080	40627200		
8	44176		
1088	40671376		
8			
1096			
8			
11040			
4			
11044			

101. **Proof.**—*To prove cube root, cube the result obtained. If the given number is an exact power, the cube of the root will equal it; if not an exact power, the cube of the root will very nearly equal it.*

102. **Rule.**—I. *Arrange the work in three columns, placing the number whose cube root is to be extracted, in the third, or right-hand, column. Begin at units place, and separate the number into periods of three figures each, proceeding from the decimal point towards the right with the decimal part, if there is any.*

II. *Find the greatest number whose cube is not greater than the number in the first period. Write this number as the first figure of the root; also, write it at the head of the first column. Multiply the number in the first column by the first figure in the root, and write the result in the second column. Multiply the number in the second column by the first figure of the root; subtract the product from the first period, and*

annex the second period to the remainder for a new dividend; add the first figure of the root to the number in the first column for the first correction. Multiply the first correction by the first figure of the root, and add the product to the number in the second column. Add the first figure of the root to the first correction to form the second correction. Annex one cipher to the second correction and two ciphers to the last number in the second column; the last number in the second column is the trial divisor.

III. *Divide the dividend by the trial divisor to find the second figure of the root. Add the second figure of the root to the number in the first column, multiply the sum by the second figure of the root, and add the result to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure of the root, subtract the result from the dividend in the third column, and annex the third period to the remainder for a new dividend. Add the second figure of the root to the number in the first column to form the first correction; multiply the first correction by the second figure of the root, and add the product to the complete divisor. Add the second figure of the root to the first correction to form the second correction. Annex one cipher to the second correction and two ciphers to the last number in the second column to form the new trial divisor.*

IV. *If there are more periods to be brought down, proceed as before. If there is a remainder after the root of the last period has been found, annex cipher periods, and proceed as before. The figures of the root thus obtained will be decimals.*

V. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

103. Art. **91** can be applied to cube root (or any other root) as well as to square root. Thus, in the example, Art. **98**, there are to be $5+1=6$ figures in the root. Extracting the root in the usual manner to $6\div 2=3$,

say 4, figures, we get for the first four figures 1,709. The last remainder is 8,556,171, and the next trial divisor with the ciphers omitted is 8,762,043. Hence, the next two figures of the root are $8{,}556{,}171 \div 8{,}762{,}043 = .976$, say .98. Therefore, the root is 1.70998.

ROOTS OF FRACTIONS.

104. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

105. EXAMPLE.—What is the square root of $\frac{9}{64}$?

SOLUTION.— $$\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}.$$ Ans.

106. EXAMPLE.—What is the square root of $\frac{5}{8}$?

SOLUTION.—Since $\frac{5}{8} = .625$, $\sqrt{\frac{5}{8}} = \sqrt{.625} = .7906$. Ans.

107. EXAMPLE.—What is the cube root of $\frac{27}{64}$?

SOLUTION.— $$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}.$$ Ans.

108. EXAMPLE.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996+$. Ans.

109. **Rule.**—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

110. Find the cube root of:

(*a*) $\frac{27}{512}$.
(*b*) 2 to five decimal places.
(*c*) 4,180,769,192.462 to five decimal places.
(*d*) $\frac{87}{125}$.
(*e*) $\frac{3}{8}$.
(*f*) 513,229.783302144 to three decimal places.

Ans.

(*a*) $\frac{3}{8}$.
(*b*) 1.25992+.
(*c*) 1,610.96238.
(*d*) .8862+.
(*e*) .7211+.
(*f*) 80.064.

TO EXTRACT OTHER ROOTS THAN THE SQUARE AND CUBE ROOTS.

111. EXAMPLE.—What is the fourth root of 256?

SOLUTION.— $\sqrt{256} = 16.$

$\sqrt{16} = 4.$

Therefore, $\sqrt[4]{256} = 4.$ Ans.

In this example, $\sqrt[4]{256}$, the index is 4, which equals 2×2. The root indicated by 2 is the square root; therefore, the square root is extracted twice.

112. EXAMPLE.—What is the sixth root of 64?

SOLUTION.— $\sqrt{64} = 8.$

$\sqrt[3]{8} = 2.$

Therefore, $\sqrt[6]{64} = 2.$ Ans.

In this example, $\sqrt[6]{64}$, the index is 6, which equals 2×3. The root indicated by 3 is the cube root; therefore, the square and cube roots are extracted in succession.

113. **Rule.**—*Separate the index of the required root into its factors (2's and 3's), and extract, successively, the roots indicated by the several factors obtained. The final result will be the required root.*

114. EXAMPLE.—What is the sixth root of 92,873,580 to two decimal places?

SOLUTION.— $6 = 3 \times 2$. Hence, extract the cube root, and then extract the square root of the result. $\sqrt[3]{92,873,580} = 452.8601$, and $\sqrt{452.8601} = 21.28+$. Ans.

115. It matters not which root is extracted first, but it is probably easier and more exact to extract the cube root first.

EXAMPLES FOR PRACTICE.

116. Extract the

(*a*) Fourth root of 100.
(*b*) Fourth root of 3,049,800,625.
(*c*) Sixth root of 9,474,296,896.

Ans. (*a*) 3.16227+.
(*b*) 235.
(*c*) 46.

RATIO.

117. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$ or .2 of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

118. A ratio may be expressed in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$, $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$, $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third is rapidly growing in favor, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is the one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, $20 \mid 4$.

119. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together they are called a **couplet**; when considered separately, the first term is called the **antecedent**, and the second term, the **consequent**. Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent, and 4, the consequent.

120. A ratio may be **direct** or **inverse**. The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal** ratio. The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20 or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

121. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

122. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5; it is the quotient obtained by dividing the antecedent by the consequent.

123. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied or both divided by the same number it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

124. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio, and dividing the antecedent or multiplying the consequent divides the ratio.

125. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

126. What is the value of the ratio of:

(*a*)	98 : 49?	(*a*)	2.
(*b*)	\$45 : \$9?	(*b*)	5.
(*c*)	$6\frac{1}{4} : \frac{3}{4}$?	(*c*)	$12\frac{1}{3}$.
(*d*)	3.5 : 4.5?	(*d*)	$.77\frac{7}{9}$.
(*e*)	The inverse ratio of 76 to 19?	(*e*)	$\frac{1}{4}$.
(*f*)	The inverse ratio of 49 to 98?	(*f*)	2.
(*g*)	The inverse ratio of 18 to 24?	(*g*)	$1\frac{1}{3}$.
(*h*)	The inverse ratio of 9 to 15?	(*h*)	$1\frac{2}{3}$.
(*i*)	The ratio of 10 to 3, multiplied by 3?	(*i*)	10.
(*j*)	The ratio of 35 to 49, multiplied by 7?	(*j*)	5.
(*k*)	The ratio of 18 to 64, divided by 9?	(*k*)	$\frac{1}{32}$.
(*l*)	The ratio of 14 to 28, divided by 5?	(*l*)	$\frac{1}{10}$.

Ans.

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127. Instead of expressing the value of a ratio by a single number as above, it is customary to express it by

means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$ or $1\frac{1}{2}:1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

128. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3}$ $= 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$, it may be done by simply dividing the exponents by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

129. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3$ $= 5^3 : 3^3$ (this expression is read, the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), and, hence, that the antecedent and consequent may both be multiplied or both divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$ or $1\frac{7}{9}:1$. Also, 4^2 $= 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9}:1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2$ $= \frac{4^2}{3^2} = 4^2 : 3^2$.

The statement may be proved for roots in the same manner. Thus $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For, the $\sqrt[3]{24^3} = 24$ and $\sqrt[3]{18^3} = 18$; and, $24:18 = 1\frac{1}{3}$ or $1\frac{1}{3}:1$. Also, $\sqrt[3]{4^3} = 4$ and $\sqrt[3]{3^3} = 3$; $4:3 = 1\frac{1}{3}$ or $1\frac{1}{3}:1$.

If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations above described cannot be performed. This is evident; for, consider the ratio of $4^2:8^3$. When expressed in the fractional form it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

PROPORTION.

130. **Proportion** is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8:4 and 6:3, which both have the same value, 2, we may employ one of the three following forms:

$$8:4 :: 6:3 \qquad (1)$$

$$8:4 = 6:3 \qquad (2)$$

$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

131. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably entirely supersede the first form. In this subject we shall adopt the second form, unless some statement can be made clearer by using the third form.

132. A proportion may be read in two ways. The old way to read the above proportion was—*8 is to 4 as 6 is to 3;* the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

133. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

134. The numbers forming the proportion are called **terms**; and they are numbered consecutively from left to right, thus:

first second third fourth

8 : 4 = 6 : 3

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

135. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

136. A **direct proportion** is one in which both couplets are direct ratios.

137. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written 8 : 4 = 6 : 3; i. e., the second ratio (couplet) must be inverted.

138. Proportion forms one of the most useful sections of arithmetic. In our grandfathers' arithmetics, it was called "The rule of three."

139. **Rule I.**—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

17 : 51 = 14 : 42.

$17 \times 42 = 51 \times 14$, since both products equal 714.

140. **Rule II.**—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion 17 : 51 = : 42?

SOLUTION.—Applying rule II, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

141. **Rule III.**—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion : 51 = 14 : 42?

SOLUTION.—Applying rule III, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans

142. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written,

$$x : 51 = 14 : 42$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

143. If the same operations (addition and subtraction excepted) be performed upon *all* the terms of a proportion, the proportion is not thereby destroyed. In other words, if all the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; (4) if the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the third form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to prove is that if any of the five operations enumerated above be performed upon all the terms of the proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since both equal $\frac{1}{2}$.

144. If both terms of either couplet be multiplied or both divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **123.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying rule II or III. Thus, the proportion $45 : 9 = x : 7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5 : 1 = x : 7.1$, whence $x = 7.1 \times 5 \div 1 = 35.5$. (See Art. **129.**)

145. The principle of all calculations in proportion is this: *Three of the terms are always given, and the remaining one is to be found.*

146. Example.—If 4 men can earn $25 in one week, how much can 12 men earn in the same time?

Solution.—The required term must bear the same relation to the given term of the same kind, as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is:

"What is it I want to find?"

In this case it is dollars. We have two sets of men, one set earning $25, and we want to know how many dollars the other set earns. It is evident that the amount 12 men earn bears the same relation to the amount 4 men earn as 12 men bear to 4 men. Hence, we have the proportion, the amount 12 men earn is to $25 as 12 men are to 4 men, or, since either extreme equals the product of the means divided by the other extreme, we have

The amount 12 men earn : $25 :: 12 men : 4 men,

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x, or the required term, occupies, the problem could be stated in any of the following forms, the value of x being the same in each:

(a) \$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(b) 4 men : 12 men = \$25 : the amount that 12 men earn; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(c) 12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

147. If the proportion is an inverse one, first form it as though it were a direct proportion, and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

148. Find the value of x in each of the following:

(a)	\$16 : \$64 :: x : \$4.	(a)	x = \$1.
(b)	x : 85 :: 10 : 17.	(b)	x = 50.
(c)	24 : x :: 15 : 40.	(c)	x = 64.
(d)	18 : 94 :: 2 : x.	(d)	$x = 10\frac{4}{9}$.
(e)	\$75 : \$100 = x : 100.	(e)	x = 75.
(f)	15 pwt. : x = 21 : 10.	(f)	$x = 7\frac{1}{7}$ pwt.
(g)	x : 75 yd. = \$15 : \$5.	(g)	x = 225 yd.

Ans.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches

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INVERSE PROPORTION.

149. In Art. **137**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example; in such cases, the proportion can

be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example, can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct, otherwise it is inverse, and one of the couplets must be inverted.

150. EXAMPLE.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5}$ = 58.8 days

EXAMPLES FOR PRACTICE.

151. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. The weight of any gas (the volume and pressure remaining the same) varies inversely as the absolute temperature. If a certain quantity of some gas weighs 2.927 lb. when the absolute temperature is 525°, what will the same volume of gas weigh when the absolute temperature is 600°, the pressure remaining the same? Ans. 2.561+ lb.

4. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 562°, what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases? Ans. 407°, very nearly.

POWERS AND ROOTS IN PROPORTION.

152. It was stated in Art. **128** that a ratio could be raised to any power or any root of it might be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power or the same root of both terms must be taken.*

153. Example.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

Solution.—This is evidently a direct proportion. Hence, we write

$$6^3 : 8^3 = x : 180.$$

Dividing both terms of the first couplet by 2^3 (see Art. **129**)

$$3^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence, $x = \frac{27 \times 180}{64} = 75\frac{15}{16}$ pounds. Ans.

Example.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\frac{15}{16}$ pounds?

Solution.—Since the weights of any two spheres are to each other as the cubes of their diameters, we have the proportion

$$180 : 75\tfrac{15}{16} = 8^3 : x^3.$$

The required term, x, must be cubed, because the other term of the couplet is cubed (see Art. **152**). But, $8^3 = 512$; hence,

$$180 : 75\tfrac{15}{16} = 512 : x^3, \text{ or } x^3 = \frac{75\frac{15}{16} \times 512}{180} = 216;$$

whence, $x = \sqrt[3]{216} = 6$ inches. Ans.

154. Since taking the same root of all the terms of a proportion does not change its value (Art. **143**), the above example might have been solved by extracting the cube root of all the numbers, thus obtaining $\sqrt[3]{180} : \sqrt[3]{75\frac{15}{16}} = 8 : x$; whence, $x = \dfrac{8 \times \sqrt[3]{75\frac{15}{16}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\dfrac{75\frac{15}{16}}{180}} = 8\sqrt[3]{\dfrac{1,215}{2,880}} = 8\sqrt[3]{\dfrac{27}{64}}$ $= 8 \times \frac{3}{4} = 6$ inches. The process, however, is longer and is not so direct, and the first method is to be preferred.

155. If two cylinders have equal volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}.$$

Since neither 12 nor 15 are perfect squares, we square all the terms (Arts. **154** and **143**) and obtain

$$81 : x^2 = 15 : 12; \text{ whence, } x^2 = \frac{81 \times 12}{15} = 64.8,$$

and $x = \sqrt{64.8} = 8.05$ inches = diameter of 15-inch cylinder.

EXAMPLES FOR PRACTICE.

156. Solve the following examples:

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 30 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet? Ans. $2\frac{1}{4}$ times as bright.

2. In the last example, suppose that the object had been 40 feet from the gas jet; how bright would it have been, compared with its brightness at 30 feet from the gas jet? Ans. $\frac{9}{16}$ as bright.

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the

source. If a man can just distinguish the time indicated by his watch, 50 feet from a certain light, at what distance could he distinguish the time from a light 3 times as powerful? Ans. 86.6+ feet.

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot?

Ans. 68,034 cu. ft. per min., nearly.

5. In the last example, suppose that 70,000 cubic feet per minute had been required; what would be the pressure necessary for this quantity? Ans. 3.81+ lb. per sq. ft.

CAUSES AND EFFECTS.

157. Many examples in proportion may be more easily solved by using the principle of *cause and effect.* That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called a **cause,** and the change or alteration, or thing accomplished, as the **effect.**

158. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words the first cause is to the second cause as the first effect is to the second effect. Thus, in the question—if 3 men can lift 1,400 pounds, how many pounds can 7 men lift?—we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause, and x pounds is the second effect. Hence, we may write

1st cause		*2d cause*		*1st effect*		*2d effect*
3	:	7	=	1,400	:	x

whence $x = \dfrac{7 \times 1{,}400}{3} = 3{,}266\frac{2}{3}$ pounds.

159. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

160. All the cases of proportion so far considered have been cases of **simple proportion**; i. e., each term has been composed of but one number. There are many cases, however, in which two or all the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects or all four consist of more than two numbers. We will illustrate this by an

EXAMPLE.—If 40 men earn $1,280 in 16 days, how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn something, 16 days is also a cause. For the same reason 36 men and 31 days are also causes. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. $1,280 is the first effect, and $\$x$ is the second effect. Hence, we write

1st cause		*2d cause*		*1st effect*		*2d effect*
40	:	36	=	1,280	:	x
16		31				

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. **118**), and the above becomes

$$\begin{matrix} 40 \\ 16 \end{matrix} \Big| \begin{matrix} 36 \\ 31 \end{matrix} = 1{,}280 \Big| \; x.$$

In the last expression, the product of all the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or } x = \frac{36 \times 31 \times \overset{2}{\cancel{1280}}\overset{80}{}}{\underset{1}{\cancel{40}} \times \underset{1}{\cancel{16}}} = \$2{,}232. \quad \text{Ans.}$$

161. The above might have been solved by canceling factors of the numbers in the original proportion. For, it any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus,

$$\begin{matrix} \cancel{40} \\ \cancel{16} \end{matrix} \Big| \begin{matrix} 36 \\ 31 \end{matrix} = \begin{matrix} 2 \\ \cancel{80} \\ \cancel{1280} \end{matrix} \Big| \; x.$$

16 is contained in 1,280, 80 times. Cancel 16 and 1,280, and write 80 above 1,280. 40 is contained in 80, 2 times. Cancel

40 and 80, and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 = \$2,232$, the same result as was obtained in the preceding article.

162. Rule.—*Write all the numbers forming the first cause in a vertical column, and draw a vertical line; on the other side of this line write in a vertical column all the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write for a fourth column, the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.*

163. Example.—If 40 men can dig a ditch 720 feet long, 5 feet wide, and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide could 24 men dig in the same time?

Solution.—Here 40 men and 24 men are the causes, and the two ditches are the effects. Hence,

$$\begin{array}{c|c|c} & \cancel{3} & \\ & \cancel{18} & \\ & \cancel{720} & x \\ \cancel{40} & 24 = \quad 5 & \cancel{3} \\ & 4 & \cancel{6} \end{array} \text{ whence, } x = 24 \times 5 \times 4 = 480 \text{ feet. Ans.}$$

164. Example.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

Solution.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|c|c|c|c} 10^2 & 16^2 & & 100 & 256 \\ 20 & 24 = 1,570.8 & x, \text{ or } & \cancel{20} & \cancel{24} = 1,570.8 & x; \\ & & & 5 & 6 \end{array}$$

whence, $x = \dfrac{256 \times 6 \times 1,570.8}{5 \times 100} = 4,825.4976$ cubic inches. Ans.

165. EXAMPLE.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what will be the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, we have

$$\begin{array}{c|c|c} 8 & 4 & \\ 5 & 12 & \\ 3 & 8 = 7{,}200 & x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds.} \quad \textbf{Ans.} \\ & 5 & \end{array}$$

166. EXAMPLE.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines the effects. Hence,

$$\begin{array}{c|c|c|c} 3 & 3 & 5 & 2 \\ 12 & 18 & 25 & 64 \\ & & 4 & \\ 30 & x = & 16 & 12, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days.} \quad \textbf{Ans.} \\ 6 & & & 4 \\ 10 & 8 & 32 & 40 \\ & & & 5 \end{array}$$

167. In examples stated like that in Art. **164**, should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose from one side of the vertical line to the other side those numbers which are said to vary inversely.

EXAMPLE.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity, and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect, we have

$$\begin{array}{c|c|c} 15 & 15 & \\ 20^2 & 24^2 = 187 & x. \\ 12 & 18 & \end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c|c} 15 & 15 & \\ & 2 & \\ 25 & 36 = 187 & x, \text{ or } x = \dfrac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds.} \quad \textbf{Ans.} \\ 400 & 576 & \\ 18 & 12 & \end{array}$$

EXAMPLES FOR PRACTICE.

168. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each?

Ans. 36 rods.

2. If a piece of iron 7 feet long, 4 inches wide, and 6 inches thick weighs 600 pounds, how much will a piece of iron weigh that is 16 feet long, 8 inches wide, and 4 inches thick? Ans. 1,828$\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide, and 8 feet high, working 8 hours a day? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed, and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch? Ans. 204.8 horsepower.

5. Referring to the example in Art. **164**, what will be the volume of a cylinder 20 inches in diameter and 24 inches long?

Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans. 45,360.

FORMULAS.

1. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

2. Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

3. As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities"; the word quantity is more general in its meaning than the word number.

4. The signs used in formulas are the ordinary signs indicative of operations, and the signs of aggregation. All these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

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5. The signs indicative of operations are six in number; viz., $+$, $-$, $\times$, $\div$, $|$, $\surd$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 / 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\surd$) indicates that some root of the quantity to the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\surd 25$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

6. The signs of aggregation are four in number; viz., $\overline{\quad}$, $(\,)$, $[\,]$, $\{\,\}$, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8}\times 7$, $(5+8)\times 7$, $[5+8]\times 7$, $\{5+8\}\times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operation.

7. While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{(5+8)}$, $\sqrt[3]{[5+8]}$, or $\sqrt[3]{\{5+8\}}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed

(except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5+8)\times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5)\div 3]\times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20-5}\div 3)\times 9-21]\div 8\}\times 12$.

8. As stated in arithmetic, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2+3\times 4$ is equal to 14, 3 being multiplied by 4, before adding to 2. Similarly, $10\div 2\times 5$ is equal to 1, since 2×5 equals 10, and $10\div 10$ is equal to 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5-9\div 3$ is equal to 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

9. There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be

performed ; it is (=), and is called the sign of *equality ;* it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5-3 = 2$, $5\times(14-9) = 25$.

10. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well known rule for finding the horsepower of a steam engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \frac{\text{mean effective pressure (in pounds per square inch)} \times \text{stroke (in feet)} \times \text{area of piston (in square inches)} \times \text{number of strokes (per minute)}}{33{,}000}.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P\times L\times A\times N}{33{,}000},$$

a much simpler and shorter expression. The last expression is called a *formula.*

11. The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent, with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33{,}000}.$$

The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1{,}006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3\,(b+c)$, $(b+c)\,a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{\overline{b+c}} \times a$, etc. are always written as here printed.

12. Before proceeding further, we will explain one other device that is used by formula makers, and which is likely to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names

of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further, and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned, when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam, at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure, and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure, or the initial absolute pressure. Second, by the use of sub. figures: thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters: thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case primes and subs. are used instead.

The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^n, etc., as they are liable to be mistaken for exponents.

13. The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body; etc.

14. It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2\times.0951\times80 + 7.8\times1\times80 + 3\frac{1}{4}\times.1138\times780}{2\times.0951 + 7.8\times1 + 3\frac{1}{4}\times.1138}$$

$$= \frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

15. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which do not require the use of logarithms for their solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\dfrac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\dfrac{160}{660}}{25}$. In every case, the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7+\dfrac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\dfrac{660}{25}}=\frac{160}{\dfrac{175+660}{25}}=\frac{160}{\dfrac{835}{25}}.$$

16. Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37 by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may

be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1+a^2b)^3}$ may also be written $(1+a^2b)^{\frac{3}{5}}$, a much simpler expression.

17. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r-h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
and h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

SOLUTION.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r-h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5-1.91)$$
$$= 40.968 - 27.95 = 13.018 \text{ square inches, nearly.} \quad \text{Ans.}$$

18. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then, substituting in the formula,

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50}\right)^2}$$

$$= \frac{50}{2}\sqrt{441 - \left(\frac{441 + 2{,}500 - 2{,}116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$

$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$

$$= 25 \times 19.312 = 482.8 \text{ square feet, nearly.} \quad \text{Ans.}$$

19. The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

20. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;

S = ultimate strength (in pounds per square inch) of the material composing the column;

A = area of cross-section of column in square inches;

q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;

l = length of column in inches;

and G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet; and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and for flat-ended steel columns, $q = \frac{1}{25{,}000}$; A, the area of the cross-section, $= .7854\,(d_1^2 - d_2^2)$ $= .7854\,(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula,

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}}$$

$$= \frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ pounds.} \quad \textbf{Ans.}$$

21. EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

(*a*) $E = \sqrt[3]{\frac{BCD}{A\left(2 + \frac{D^2}{C^2}\right)}}$? (*b*) $E = \frac{A - \frac{3}{4}D + \frac{4B^2}{A + C}}{A - \sqrt{\frac{2B^2}{A + 22}}}$?

SOLUTION.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have,

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}$$

Reducing the fraction to a decimal, so that it will be easier to extract the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \quad \text{Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}}$$

$$= \frac{7 + 17.066 +}{10 - \sqrt{4}} = \frac{24.066 +}{8} = 3.008 +. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \dfrac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \dfrac{\frac{2}{3}(A + e)}{c\,e}$. Ans. $x = 1\frac{1}{3}$.

3. $x = \sqrt{\dfrac{d^2}{2c}} + \sqrt{A\,B^2}$. Ans. $x = 29$.

4. $x = \dfrac{A\,e}{\sqrt{16\,B\,c}} + \dfrac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2\,e)\left(\sqrt[3]{B} - \dfrac{1}{c}\right) + \dfrac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{13}$.

6. $x = \sqrt{\dfrac{B\,c\,d}{A\left(2 + \frac{d^3}{e^2}\right)}}$. Ans. $x = .396+$.

GEOMETRY AND MENSURATION.

GEOMETRY.

LINES AND ANGLES.

1. **Geometry** is that branch of mathematics which treats of the properties of lines, angles, surfaces, and volumes.

2. A **point** indicates position only. It has neither length, breadth, nor thickness.

3. A **line** has only one dimension: length.

4. A **straight line** is one that does not change its direction throughout its whole length. See Fig. 1. A straight line is also frequently called a **right line.**

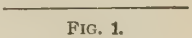

FIG. 1.

5. A **curved line** changes its direction at every point. See Fig. 2.

FIG. 2.

6. A **broken line** is one made up wholly of straight lines lying in different directions. See Fig. 3.

FIG. 3.

7. **Parallel lines** are equally distant from each other at all points. The lines shown in Fig. 4 are parallel.

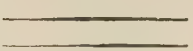

FIG. 4.

8. A line is **perpendicular to** another when it meets that line so as not to incline towards it on either side. Thus, in Fig. 5, the line denoted by the letters *A B* is perpendicular to that denoted by *C D*.

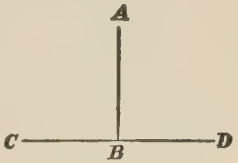

FIG. 5.

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FIG. 6.

9. A **horizontal line** is a line parallel to the horizon, or water level. See Fig. 6.

10. A **vertical line** is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb-line. See Fig. 6.

11. When two lines cross or cut each other, they are said to **intersect,** and the point at which they intersect, as *A*, Fig. 7, is called the **point of intersection.**

FIG. 7.

FIG. 8.

12. An **angle** is the opening between two lines which intersect, or meet; the point of meeting is called the **vertex** of the angle. See Fig. 8.

13. In order to distinguish one line from another, two of its points are given if it is a straight line, and as many more as are considered necessary if it is a broken or curved line. Thus, in Fig. 9, the line *A B* would mean the straight line included between the points *A* and *B*. Similarly, the straight line between *C* and *B*, or between *B* and *D*, would be called the line *C B*, or the line *B D*. The broken line made up of the lines *A B* and *C B*, or *B D*, would be called the broken line *C B A* or *A B C*, and *A B D* or *D B A*, according to the point started from.

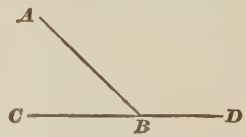

FIG. 9.

To distinguish angles, a point on each line and the point of their intersection, or vertex of the angle, are named; thus, in Fig. 9, the angle formed by the lines *A B* and *C B* is called the angle *A B C* or the angle *C B A*; the letter at the vertex is always placed in the middle. The angle formed by the lines *A B* and *B D* is called the angle *A B D* or the angle *D B A*.

When an angle stands alone so that it cannot be mistaken

for any other angle, only the vertex letter need be given; thus, the angle O, or the angle P, etc.

14. If one straight line meets another straight line at a point between its ends, two angles, $A\,B\,C$ and $A\,B\,D$, Fig. 9, are formed, which are called **adjacent angles.**

15. When these adjacent angles, $A\,B\,C$ and $A\,B\,D$, are equal, they are called **right angles.** See Fig. 10.

FIG. 10.

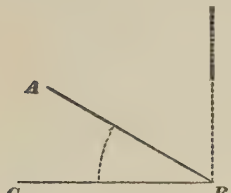

FIG. 11.

16. An **acute angle** is less than a right angle. Thus, $A\,B\,C$, Fig. 11, is an acute angle.

17. An **obtuse angle** is greater than a right angle. The angle $A\,B\,D$, Fig. 12, is an obtuse angle.

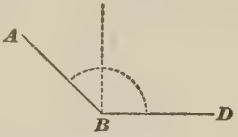

FIG. 12.

18. When two straight lines intersect, they form four angles about the point of intersection. Thus, in Fig. 13, the lines $A\,B$ and $C\,D$, intersecting at the point O, form four angles, $B\,O\,D$, $D\,O\,A$, $A\,O\,C$, and $C\,O\,B$, about the point O. The angles which lie on the *same* side of one straight line, as $D\,O\,B$ and $D\,O\,A$ are **adjacent angles.** The angles which lie *opposite* each other are called **opposite angles.** Thus, $A\,O\,C$ and $D\,O\,B$, also $D\,O\,A$ and $B\,O\,C$, are opposite angles.

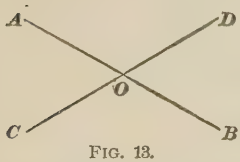

FIG. 13.

When one straight line intersects another straight line, as in Fig. 13, the opposite angles are equal. Thus, $D\,O\,B = A\,O\,C$, and $D\,O\,A = B\,O\,C$.

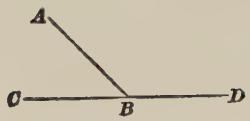

FIG. 14.

19. When one straight line meets another straight line at a point between its ends, the sum of the two adjacent angles, as ABD and ABC, Fig. 14, is equal to two right angles.

20. If a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles. Thus, in Fig. 15, $COB + DOC + EOD + FOE + AOF$ = two right angles.

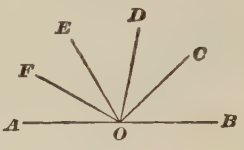

FIG. 15.

21. If a straight line intersects another straight line, so that the adjacent angles are equal, the lines are said to be **perpendicular** to each other. In such a case, four right angles are formed about the point of intersection. Thus, in Fig. 16, $BOC = COA$; hence, BOC, COA, AOD, and DOB are right angles. From this, it is seen that *four right angles* are all that can be formed about a given point.

FIG. 16.

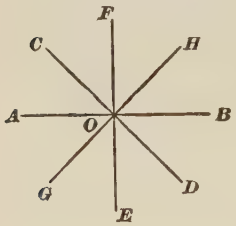

FIG. 17.

It follows that, if through a given point, any number of straight lines are drawn, the sum of all the angles formed about the point of intersection is equal to four right angles. Thus, in Fig. 17, $HOF + FOC + COA + AOG + GOE + EOD + DOB + BOH$ = four right angles.

EXAMPLE.—A circular window has 12 ribs equally spaced. What part of a right angle is included between the center lines of any two ribs?

SOLUTION.—Since there are 12 ribs, there are 12 angles. The sum of all the angles equals four right angles. Hence, one angle equals $\frac{1}{12}$ of four right angles, or $\frac{4}{12} = \frac{1}{3}$ of one right angle. Ans.

22. A perpendicular drawn from a point over or under a given straight line is the shortest distance from the point to the line, or to the line extended. Thus, if A, Fig. 18, is the given point, and CD, the given line, then the perpendicular AB is the shortest distance from A to CD.

FIG. 18.

23. If two angles have their sides parallel, and lie in the same or in opposite directions, they are equal. Thus, if the side AB, Fig. 19 or Fig. 20, is parallel to the side DE, and if the side BC is parallel to the side EF, then the angle E = the angle B.

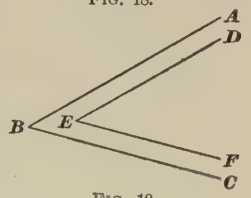

FIG. 19.

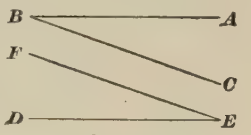

FIG. 20.

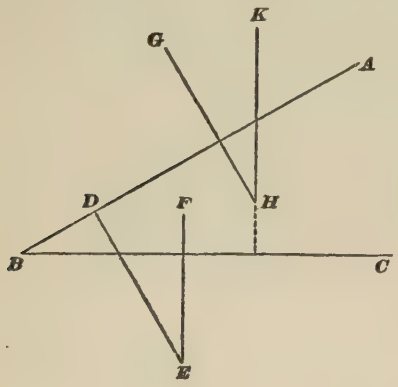

FIG. 21.

24. If two sides of an angle are perpendicular to two sides of another angle, the two angles are equal. Thus, if DE and GH, Fig. 21, are perpendicular to BA, and EF and HK are perpendicular to BC, then will angle E = angle B = angle H.

EXAMPLES FOR PRACTICE.

25. Solve the following examples:

1. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{4}{5}$ of a right angle.

2. If one straight line meets another straight line so as to form an angle equal to $1\frac{3}{5}$ right angles, what part of a right angle does its adjacent angle equal? Ans. $\frac{3}{5}$ of a right angle.

3. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, what part of a right angle is contained in each angle?

Ans. $\frac{1}{6}$ of a right angle.

PLANE FIGURES.

26. A **surface** has only two dimensions: *length* and *breadth*.

27. A **plane surface** is a flat surface. If a straightedge be laid on a plane surface, every point along the edge of the straightedge will touch the surface, no matter in what direction it is laid.

28. A **plane figure** is any part of a plane surface bounded by straight or curved lines.

29. When a plane figure is bounded by **straight** lines, it is called a **polygon.** The bounding lines are called the **sides,** and the length of the broken line that bounds it (or the whole distance around it) is called the **perimeter** of the polygon.

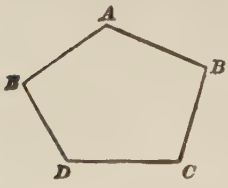

FIG. 22.

30. The angles formed by the sides are called the **angles** of the polygon. Thus, $ABCDE$, Fig. 22, is a polygon. AB, BC, etc. are the **sides**; EAB, ABC, etc. are the **angles**; and the length of the broken line $ABCDE$ is the **perimeter.**

31. Polygons are classified according to the number of their sides: One of three sides is called a **triangle**; one of four sides, a **quadrilateral**; one of five sides, a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon**; one of eight sides, an **octagon**; one of ten sides, a **decagon**; one of twelve sides, a **dodecagon**; etc.

32. **Equilateral polygons** are those in which the sides are all equal. Thus, in Fig. 23, $AB = BC = CD = DA$; hence, $ABCD$ is an equilateral polygon.

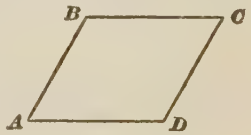

FIG. 23.

33. An **equiangular polygon** is one in which all the angles are equal. Thus, in Fig. 24, angle A = angle B = angle D = angle C; hence, $A\,B\,D\,C$ is an equiangular polygon.

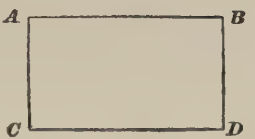

FIG. 24.

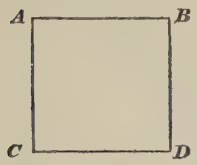

FIG. 25.

34. A **regular polygon** is one in which all the sides and all the angles are equal. Thus, in Fig. 25, $A\,B = B\,D = D\,C = C\,A$, and angle A = angle B = angle D = angle C; hence, $A\,B\,D\,C$ is a regular polygon.

Other regular polygons are shown in Fig. 26.

FIG. 26.

35. The sum of all the interior angles of any polygon is equal to two right angles multiplied by a number which is two less than the number of sides in the polygon. Thus, $A\,B\,C\,D\,E\,F$, Fig. 27, is a polygon of six sides (hexagon), and the sum of all the interior angles $A+B+C+D+E+F$ = 2 right angles $\times 4$ $(= 6-2)$, or 8 right angles.

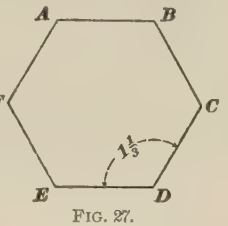

FIG. 27.

EXAMPLE.—Fig. 27 represents a regular hexagon (has equal sides and equal angles). How many right angles are there in each interior angle?

SOLUTION.—The sum of the interior angles is $2 \times (6-2) = 8$ right angles; and, as there are six equal angles, we have $8 \div 6 = 1\frac{1}{3}$ right angles, the number of right angles in each interior angle. Ans.

THE TRIANGLE.

36. Triangles may be divided, with respect to their sides, into *isosceles*, *equilateral*, and *scalene* triangles; and with respect to their angles, into *right-angled* and *oblique-angled* triangles.

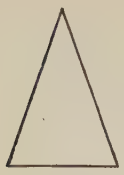

FIG. 28.

37. An **isosceles** triangle is one having two of its sides equal. See Fig. 28.

38. An **equilateral** triangle is one that has the three sides equal. See Fig. 29.

FIG. 29.

FIG. 30.

39. A **scalene** triangle is one having no two of its sides equal. See Fig. 30.

40. A **right-angled** triangle is any triangle having one right angle. See Fig. 31. The side opposite the right angle is called the **hypotenuse.** For brevity, a right-angled triangle is usually termed a **right triangle.**

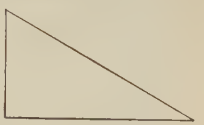

FIG. 31.

FIG. 32.

41. An **oblique-angled** or **oblique** triangle is one which has no right angles. See Fig. 32.

42. The **base** of any triangle is the side upon which the triangle is supposed to stand.

The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpendicular to the base or to the base extended. Thus, in Figs. 33 and 34, the side $A\ C$ is the base of the triangle and the line BD is the altitude.

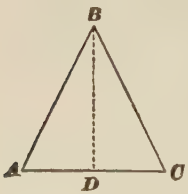

FIG. 33.

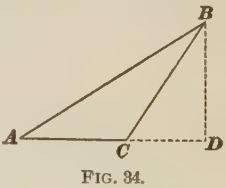

FIG. 34.

In a right triangle, if one of the short sides is taken as

the base, the other short side will be the altitude of the triangle.

43. In an isosceles triangle, the angles opposite the equal sides are equal. Thus, in Fig. 35, $A\,B = B\,C$; hence, angle C = angle A.

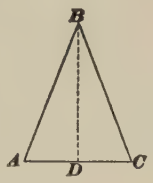

FIG. 35.

In any isosceles triangle, if a perpendicular be drawn from the vertex opposite the unequal side to that side, it bisects (cuts in halves) the side. Thus, $A\,C$ is the unequal side in the isosceles triangle $A\,B\,C$; hence, the perpendicular $B\,D$ bisects $A\,C$, or $A\,D = D\,C$.

If two angles of any triangle are equal, the triangle is isosceles.

44. In any triangle, the sum of the three angles is equal to two right angles. Thus, in Fig. 36, the sum of the angles at A, B, and C = two right angles; that is, $A + B + C$ = two right angles. Hence, if any two angles of a triangle are given, the third may be found by subtracting the sum of the two from two right angles. Suppose that $A + B = 1\frac{7}{10}$ right angles; then, C must equal $2 - 1\frac{7}{10} = \frac{3}{10}$ of a right angle.

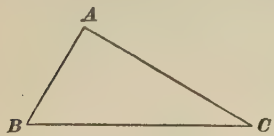

FIG. 36.

45. In any right-angled triangle there can be but one right angle, and since the sum of all the angles equals two right angles, it is evident that the sum of the two acute angles must be equal to a right angle. Therefore, if in any right-angled triangle one acute angle is known, the other can be found by subtracting the known angle from a right angle. Thus $A\,B\,C$, Fig. 37, is a right-angled triangle, right-angled at C. Then, the angles $A + B$ = one right angle. If $A = \frac{3}{7}$ of a right angle, $B = 1 - \frac{3}{7} = \frac{4}{7}$ of a right angle.

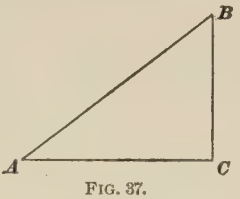

FIG. 37.

46. In any right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares

described upon the other two sides. If $A\,B\,C$, Fig. 38, is a right-angled triangle, right-angled at B, then the square described upon the hypotenuse $A\,C$ is equal to the sum of the squares described upon the sides $A\,B$ and $B\,C$; consequently, if the lengths of the sides $A\,B$ and $B\,C$ are known, we can find the length of the hypotenuse by adding the squares of the lengths of the sides $A\,B$ and $B\,C$, and then extracting the square root of the sum.

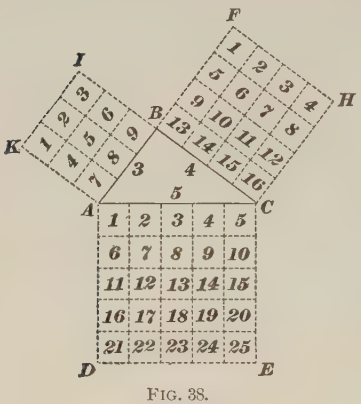

FIG. 38.

If the length of one of the short sides be denoted by a, that of the other short side by b, and that of the hypotenuse by c, then, since $c^2 = a^2 + b^2$, we have, by extracting the square root of both quantities,

$$c = \sqrt{a^2 + b^2}.$$

EXAMPLE.—The width of a house is 20 feet, and the roof is *half pitch*—that is, *the height of the gable is equal to one-half the width.* What is the length of a rafter, if it projects 15 inches over the side?

SOLUTION.—Let one-half the width be called $a = 10$ feet; the height of the gable, $b = \frac{1}{2}$ of 20, or 10 feet; and the hypotenuse, c. Substituting these values in the formula,

$$c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14 \text{ feet.}$$

Adding 15 inches = 1.25 feet, the length of a rafter is 15.39 feet; or, reducing .39 foot to inches, the length is 15 feet $4\frac{3}{4}$ inches, nearly. Ans.

47. If the hypotenuse and one side are given, the other side can be found by subtracting the square of the given side from the square of the hypotenuse, and then extracting the square root of the remainder. That is,

$$a = \sqrt{c^2 - b^2} \text{ or } b = \sqrt{c^2 - a^2},$$

the letters having the same meaning as in Art. **46.**

EXAMPLE.—It is desired to ascertain the height of the ceiling in a room. One end of a 10-foot pole is set in the corner of the ceiling and

wall, and the other end is 6 feet from the base of the wall. What is the height of the ceiling?

SOLUTION.—In this problem, the 10-foot pole is the hypotenuse c, and the short side b is 6 feet. Hence, applying the formula,

$$a = \sqrt{c^2 - b^2} = \sqrt{100 - 36} = 8 \text{ feet},$$

which is the height of the ceiling. Ans.

48. If the sides are equal, or if, in Fig. 39, $a = b$, then the hypotenuse is equal to the square root of twice the square of either side; that is, $c = \sqrt{2\,a^2} = \sqrt{2\,b^2}$. Also, $a = b = \sqrt{\frac{c^2}{2}}$; that is, either side is equal to the square root of one-half the square of the hypotenuse.

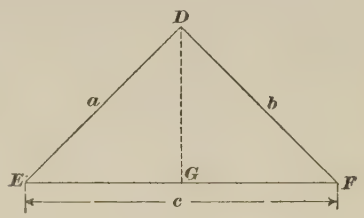

FIG. 39.

In such a triangle, the perpendicular distance from the hypotenuse to the right angle is one-half the hypotenuse. Thus, in roofing, if $G\,D$, Fig. 39, is equal to $E\,G$, or $\frac{1}{2}\,E\,F$, then the roof is called *half pitch.*

EXAMPLE.—What is the length of a slope of a roof having half pitch, the width of the house being 30 feet?

SOLUTION.—In this example, $c = 30$ feet, and $a = b$. Then, by the formula,

$$a = b = \sqrt{\frac{c^2}{2}} = \sqrt{450} = 21.21 \text{ feet}. \quad \text{Ans.}$$

EXAMPLE.—It is desired to lay out a line $A\,C$ at right angles to a line $A\,B$, which is 15 feet long. How can it be done by means of a tape?

SOLUTION.—If we can find any two numbers, the sum of whose squares is equal to the square of another number, these three numbers will be the sides of a right triangle; as, for example, 3, 4, and 5, or 6, 8, and 10. Thus, hold the 6-foot mark on the tape at A, Fig. 40, the 14-foot mark at D, and bring the end of the tape and the 24-foot mark together at C, and mark the point C, then $A\,C$ will be perpendicular to $A\,D$; for $\sqrt{A\,C^2 + A\,D^2} = C\,D$; that is, $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10$ feet $= 24$ feet $- 14$ feet.

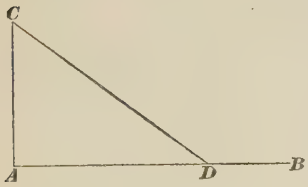

FIG. 40.

49. The principle of the right triangle is of very great value in practical work, and the student should become thoroughly familiar with it in all its variations.

The following is an example showing a double application of the right-triangle principle:

EXAMPLE.—In Fig. 41, $A\,B\,C\,D$ represents a skylight 7 ft. 6 in. $\times$ 9 ft. The point O is 2 feet above the plane of $A\,B\,C\,D$. What is the length of the hip rafter $O\,D$ at the angle of the skylight?

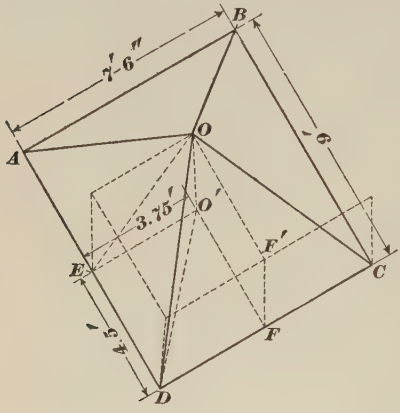

FIG. 41.

SOLUTION.—It will be seen that the point O' is directly under O and 2 feet below it, so that O' is on the same level as $A\,B\,C\,D$. It will also be seen that $D\,O$ is simply the hypotenuse of a right triangle, whose base is $D\,O'$ and whose altitude is $O\,O'$. But $D\,O'$ is also the hypotenuse of a right triangle, whose sides are $E\,O'$ and $E\,D$. $E\,O' = \frac{1}{2}$ of 7 feet 6 inches $= 3.75$ feet, and $E\,D = \frac{1}{2}$ of 9 feet $= 4.5$ feet. Then, $D\,O' = \sqrt{(3.75)^2 + (4.5)^2} = 5.86$ feet. In the triangle $D\,O'\,O$, $D\,O' = 5.86$ feet, and $O\,O' = 2$ feet; whence,

$$D\,O = \sqrt{(5.86)^2 + 2^2} = 6.19 \text{ feet} = 6 \text{ feet } 2\tfrac{1}{4} \text{ inches, nearly.}$$

One extraction of square root may be dispensed with, thus:

$$D\,O = \sqrt{(D\,O')^2 + (O\,O')^2} = \sqrt{[(E\,O')^2 + (E\,D)^2] + (O\,O')^2}$$

$$= \sqrt{(3.75)^2 + (4.5)^2 + 2^2} = \sqrt{38.31} = 6.19 \text{ feet, as before.} \quad \text{Ans.}$$

Again, if we can find the length of a line along the skylight perpendicular to an edge, as $O\,E$ to $A\,D$, then the hip $O\,D$ is the hypotenuse of a right triangle of which $O\,E$ and $E\,D$ are sides; and $O\,E = \sqrt{(O'\,E)^2 + (O\,O')^2}$.

The result will be the same either way.

NOTE.—When decimals occur in solving the examples in this section on *Geometry and Mensuration*, two decimal places are to be retained (except in constants, as .7854, .5236, etc.), and if the third decimal figure is *greater* than 5, the second decimal figure is to be increased by 1; thus, 13.537 should be written 13.54, not 13.53. To easily convert feet and inches to feet and decimals of a foot, feet and decimals of a foot to feet and inches, or to change a fraction of an inch to a decimal, or vice versa, the conversion tables, Art. **82**, may be used.

SIMILAR TRIANGLES.

50. Two triangles are **equal** when the *sides* of one are equal to the sides of the other.

51. Two triangles are **similar** when the *angles* of one are equal to the angles of the other. *The corresponding sides of similar triangles are proportional.*

For example, suppose we have two triangles $A\,B\,C$ and $a\,b\,c$, Fig. 42, in which the side $a\,c$ is perpendicular to $A\,C$, the side $a\,b$, to $A\,B$, and side $c\,b$, to $B\,C$, then, angle A = angle a, since the sides of one are perpendicular to the sides of the other. (See Art. **24.**) In like manner, angle B = angle b, and angle C = angle c. The two triangles are therefore similar, and their corresponding sides are proportional. That is, any two sides of one triangle are to each other as the two corresponding sides of the other triangle; or, one side of one triangle is to the corresponding side of the other as another side of the first triangle is to the corresponding side of the second. The following are examples of the many proportions that may be written. In this case, the corresponding sides of the two triangles are the ones perpendicular to each other.

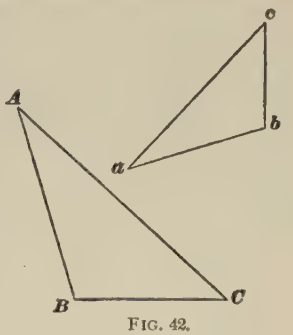

FIG. 42.

$$A\,B : B\,C = a\,b : b\,c, \qquad B\,C : b\,c = A\,B : a\,b,$$
$$A\,B : A\,C = a\,b : a\,c, \qquad A\,C : a\,c = B\,C : b\,c, \text{ etc.}$$

EXAMPLE.—It is required to find the distance $A\,B$, Fig. 43, across a stream.

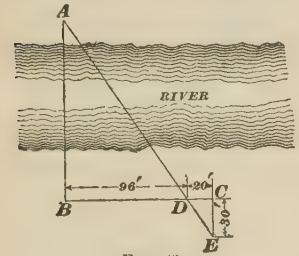

FIG. 43.

SOLUTION.—The line $B\,C$ making any angle with $A\,B$ is measured, and $C\,E$ is made parallel with $A\,B$, and of any convenient length. The point D is marked where $A\,E$ intersects $B\,C$, and $B\,D$ and $D\,C$ are measured. Then since the triangle $A\,B\,D$ and $C\,D\,E$ have their corresponding sides parallel, they are similar, and $A\,B : C\,E = B\,D : D\,C$; or $A\,B = \frac{30 \times 96}{20} = 144$ feet. Ans.

52. If a straight line be drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally. Thus, let the line DE be drawn parallel to the side BC in the triangle ABC, Fig. 44. Then, $AD:AB = AE:AC$, or $AD:DB = AE:EC$. It is to be noticed that the triangles ADE and ABC are similar, and their sides are proportional. Thus $AB:AD = BC:DE$ and $AC:AE = BC:DE$.

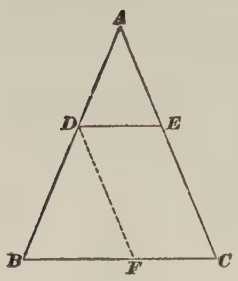

FIG. 44.

If a straight line, as DF, be drawn from D or E, Fig. 44, parallel to AC or AB, then the triangles ADE, ABC, and DBF are all similar, and a number of other proportions may be formed.

EXAMPLE.—Referring to Fig. 45, it is desired to find the length of the line CA, extending across a river. E is on the line AC, D is on the line BA, and DE is parallel to CB; the lengths of CE, ED, and CB are as shown in the figure.

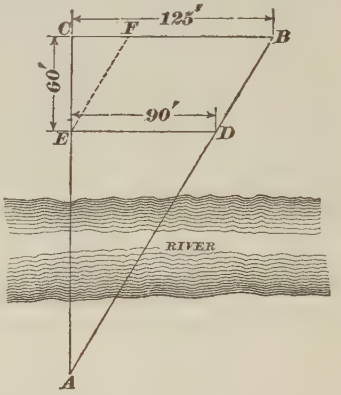

FIG. 45.

SOLUTION.—Draw EF parallel to AB, so that $CF = 125 - 90 = 35$ feet; then $CA:CE = CB:CF$, or $CA:60 = 125:35$, whence, $CA = \frac{60 \times 125}{35}$ $= 214.3$ feet, nearly.

EXAMPLE.—On a drawing it is required to divide a line 8 inches long into 12 parts.

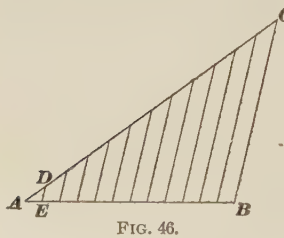

FIG. 46.

SOLUTION.—Let AB, Fig. 46, be the 8-inch line. Through A draw any line AC 12 inches long, and mark the inch points on it. Connect C and B, and through each inch mark on AC, draw a parallel to CB, as DE, etc., cutting AB at E, etc., which will be the points required; for by similar triangles, $AE:AD = AB:AC$; or AE:1 in. = 8 in.:12 in.; hence, $AE = \frac{1 \times 8}{12} = \frac{2}{3}$ inch. Ans.

This principle is very useful when an exact measurement—as for example, $\frac{2}{3}$ inch—cannot be obtained by a scale or rule.

EXAMPLES FOR PRACTICE.

53. Solve the following:

1. The distance from the first to the second floor of a house is 9 feet. It is desired to mark the line of the top of the 14 steps, or *treads*, on a drawing. What is the height of each *riser?* Make a sketch showing how the spacing may be found. Ans. 7.7 in. $= 7\frac{11}{16}$ in., nearly.

2. What length of stone coping is required for each side of a gable, the width of which is 24 feet, and whose height is $\frac{3}{4}$ the span? Ans. 21.63 ft.

3. What is the angle at the top of a gable of a house whose roof slopes are each one-half of a right angle? Ans. A right angle.

4. In running a line FA, Fig. 47, a house stands in the way. AC is laid off square with AF, 14 feet long, and 12 feet behind AC, a line GE 23 feet long is laid off at right angles to AF, so as to give a sight past the house to B. What are the lengths of AB and CB? $CE = \sqrt{9^2 + 12^2}$. Ans. $\begin{cases} AB, 18\frac{2}{3} \text{ ft.} \\ CB, 23\frac{1}{3} \text{ ft.} \end{cases}$

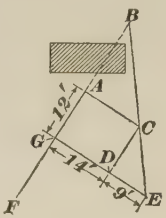

FIG. 47.

5. What is the distance between the 16-inch mark on one leg of a carpenter's square and the 12-inch mark on the other? Ans. 20 in.

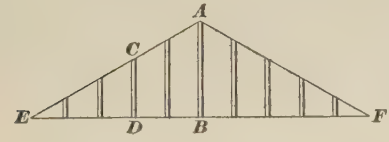

FIG. 48.

6. The distance EB, Fig. 48, is $8\frac{1}{3}$ feet. If the spacing of the roof rafters is 20 inches, and the true length of AB is 12 feet, what is the length of the "jack rafter" CD? Ans. $7\frac{1}{5}$ ft. $= 7$ ft. $2\frac{1}{2}$ in., nearly.

7. In Fig. 49, $CBAEC$ represents a gable at right angles to the main roof, the line AB, where they meet, being called a *valley*. What is the length of the valley rafter AB, CA being 16 feet, ED, the *rise* of the rafter, being 8 feet, and $EB = DF$, the distance of B back of ACE, 4 feet? The principle is the same as explained for *hips*. Ans. 12 ft.

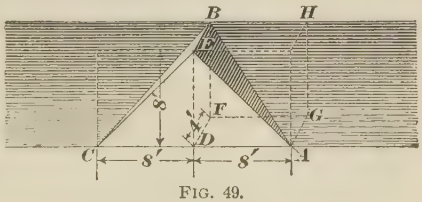

FIG. 49.

THE CIRCLE.

FIG. 50.

54. A **circle** is a plane figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.** See Fig. 50.

55. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference, as $A\,B$, Fig. 51.

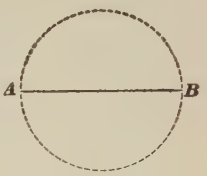

FIG. 51.

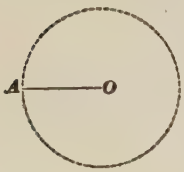

FIG. 52.

56. The **radius** of a circle, $O\,A$, Fig. 52, is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of the word radius is **radii.** All radii of any circle are equal in length.

57. An **arc** of a circle is any part of its circumference, as $A\,E\,B$, Fig. 53.

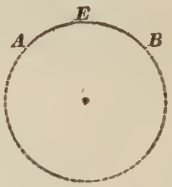

FIG. 53.

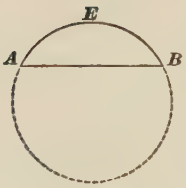

FIG. 54.

58. A **chord** is a straight line joining any two points in a circumference; or, it is a straight line joining the extremities of an arc. Thus, in Fig. 54, $A\,B$ is the chord of the arc $A\,E\,B$.

59. A **segment** of a circle is the space included between the arc and its chord. Thus, in Fig. 54, the space between the arc $A\,E\,B$ and the chord $A\,B$ is a segment.

60. A **sector** of a circle is the space included between an arc and two radii drawn to the extremities of the arc, as *A O B*, Fig. 55.

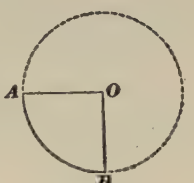

FIG. 55.

61. Two circles are equal when the *radius or diameter* of one is equal to the *radius or diameter* of the other.

Two arcs are equal when the *radius and chord* of one is equal to the *radius and chord* of the other.

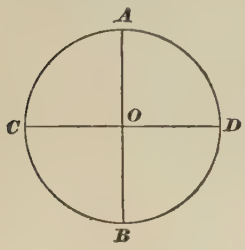

FIG. 56.

62. If *A D B C*, Fig. 56, is a circle in which two diameters *A B* and *C D* are drawn at right angles to each other, then *A O D*, *D O B*, *B O C*, and *C O A* are right angles. The circumference is thus divided into four equal parts; each of these parts is called a **quadrant.**

63. To measure angles, the circumference of circles are divided into 360 equal parts called *degrees*, which are subdivided into 60 equal parts called *minutes;* and the latter are further subdivided into 60 equal parts called *seconds.* Degrees, minutes, and seconds are indicated by the marks °, ′, ″; thus, 65 degrees, 15 minutes, and 40 seconds is written 65° 15′ 40″. Since a quadrant is one-fourth of a circumference, it includes $\frac{1}{4}$ of 360°, or 90°, whence a right angle contains 90°. So, also, if a circle be divided into equal sectors, the angle included between two adjacent radii is equal to 360° divided by the number of sectors. Thus, the angle between radii drawn to the angles of a regular octagon includes $\frac{1}{8}$ of 360°, or 45°.

The intersection of any two straight lines may be considered the center of a circle, and the number of 360ths, or degrees—measured on any circumference described from

this center—included between the lines, measures the angle.

64. An **inscribed** angle is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by *one-half* the intercepted arc. Thus, in Fig. 57, $A\,B\,C$ is an inscribed angle, and it is measured by one-half the arc $A\,D\,C$.

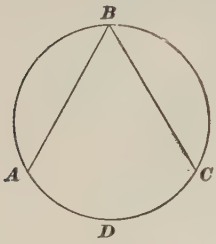

FIG. 57.

EXAMPLE.—If, in Fig. 57, the arc $A\,D\,C = \frac{2}{5}$ of the circumference, how many degrees are there in the inscribed angle $A\,B\,C$?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference contains 360°; and $360° \times \frac{1}{5} = 72°$. Ans.

65. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.** The measure of a semicircle is one-half of 360°, or 180°.

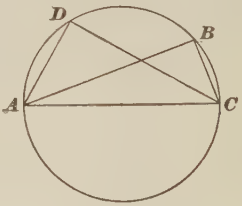

FIG. 58.

Any angle that is inscribed in a semicircle and intercepts a semi-circumference, as $A\,B\,C$ or $A\,D\,C$, Fig. 58, is a right angle, since it is measured by one-half a semi-circumference, or by 90°.

66. If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\,C$, Fig. 59, is perpendicular to the chord $A\,B$, $A\,D = D\,B$.

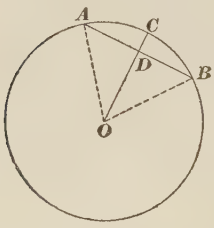

FIG. 59.

A radius which bisects a chord, bisects also the angle included between radii drawn to the ends of the chord. Thus, in Fig. 59, the radius $O\,C$ bisects the angle $A\,O\,B$.

If a straight line be drawn perpendicular to any chord at its middle point, it must pass through the center of the circle.

67. Through any three points not in the same straight line, a circumference can be drawn. Let A, B, and C, Fig. 60, be any three points. Join A and B, and B and C by straight lines. At the middle point of AB, draw HK perpendicular to AB; at the middle point of BC draw EF perpendicular to BC. These two perpendiculars intersect at O. With O as a center, and OB, OA, or OC as a radius, describe a circle; it will pass through A, B, and C.

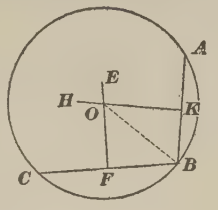

FIG. 60.

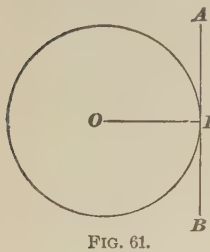

FIG. 61.

68. **A tangent** to a circle is a straight line which touches the circle at one point only; it is always perpendicular to a radius drawn to that point. Thus, AB, Fig. 61, is a tangent to the circle; it touches the circle at E and is perpendicular to the radius OE.

69. If two circles intersect each other, the line joining their centers bisects at right angles the line joining the two points of intersection. Thus, if the two circles, whose centers are O and P, Fig. 62, intersect at A and B, the line OP bisects at right angles the line AB; or $AC = BC$.

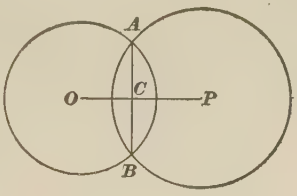

FIG. 62.

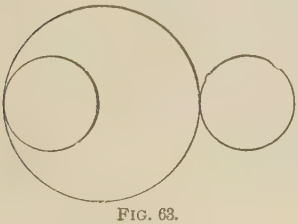
FIG. 63.

70. One circle is said to be **tangent** to another circle when they touch each other at one point only. See Fig. 63. This point is called the **point of contact,** or the **point of tangency.**

71. When two or more circles are described from the same center, they are called **concentric circles.** See Fig. 64.

FIG. 64.

72. If, from any point on the circumference of a circle, a perpendicular be let fall upon a given diameter, this perpendicular will be a mean proportional between the two parts into which it divides the diameter.

If AB, Fig. 65, is the diameter, and C any point on the circumference, then the perpendicular CD is a mean proportional between AD and DB, or $AD : CD = CD : DB$. Therefore, $\overline{CD}^2 = AD \times DB$, and $CD = \sqrt{AD \times DB}$. This principle furnishes a method of drawing a mean proportional between two lines. Let AD and DB, Fig. 65, be any two lines. Join them together in one line as AB, and on this as a diameter, draw a circle. Then CD, perpendicular to this diameter at the common end D, is the mean proportional.

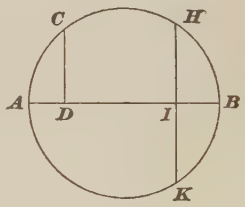

FIG. 65.

EXAMPLE.—In arches the *span* is the distance across the opening, as AC, Fig. 66, measured from the ends of the arch as A and C. The *rise* DB is the perpendicular distance from AC to the highest point B measured on the center line OB; DB = the radius $- OD$.

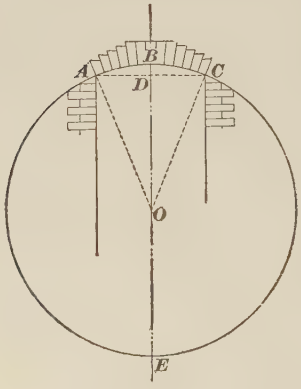

FIG. 66.

The span AC, Fig. 66, of an arch is 6 feet, and the rise DB is 8 inches. What is the length of the radius OB?

SOLUTION. — Here the span is the chord of a circle, and since a radius perpendicular to a chord bisects it, AD is $\frac{1}{2}$ of AC = 36 inches. If we draw the complete circle, it will be seen that ED and DB are the parts of the diameter and AD the perpendicular from the point A. Then $\overline{AD}^2 = ED \times DB$, or $36^2 = ED \times 8$, whence, $ED = 1{,}296 \div 8 = 162$ inches, and $EB = 162 + 8 = 170$ inches. $OB = \frac{1}{2} EB = 85$ inches, or 7 feet 1 inch. Ans.

INSCRIBED AND CIRCUMSCRIBED POLYGONS.

73. An **inscribed** polygon is one whose vertices lie on the circumference of a circle and whose sides are chords, as $KLMNPQ$, Fig. 67.

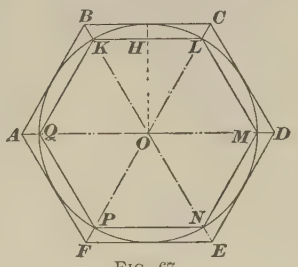

FIG. 67.

74. A **circumscribed** polygon is one whose sides are tangent to a circle, as $ABCDEF$, Fig. 67.

Circles may be inscribed in and circumscribed about any regular polygon. The radius of a circle is the distance from the center to an angle of the inscribed polygon, and the perpendicular distance from the center to the side of the circumscribed polygon, as shown in Fig. 67.

75. If lines be drawn from the center to the angles of a regular polygon, they will make equal angles with the sides of the polygon and will form isosceles triangles. As the sum of all the angles formed by lines drawn from a point is 4 right angles, or 360°, the angle at the center of a regular polygon between two radii equals 360° divided by the number of sides. Thus the angle between two radii drawn to the end of a side of a regular hexagon is $360° \div 6 = 60°$. The sum of the angles of a triangle is 2 right angles, or 180°; hence, since the triangles formed by drawing radii to the angles of a regular polygon are isosceles, the angles OAB and OBA, Fig. 67, are each equal to $\frac{1}{2}$ the difference between 180° and the central angle. Thus, each angle $= \frac{1}{2} \times (180° - 60°) = 60°$, and the triangle OBA is equilateral. In a regular hexagon, therefore, the sides are equal to the radius of the circumscribed circle.

A line drawn from the center of a regular polygon to an angle bisects the angle; thus OA, Fig. 67, bisects the angle BAF, which, therefore, is equal to $2 \times OAB$, or $2 \times OAF$.

EXAMPLE.—As the principles of Art. **75** are used in forming bevel angles, etc., let us find at what angle the bevel must be set to mark the cuts for the moldings around an octagonal room of equal sides, as in Fig. 68.

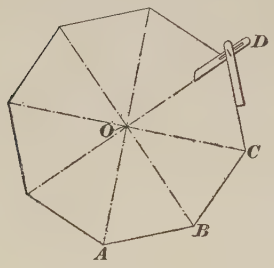

FIG. 68.

SOLUTION.—Draw $A\,O$, $B\,O$, etc. to the center O, forming triangles $A\,O\,B$, $B\,O\,C$, etc. Then each of the angles at O is $\frac{1}{8}$ of $360° = 45°$. The angles $O\,A\,B$ and $O\,B\,A$ are each $\frac{1}{2} \times (180° - 45°) = 67\frac{1}{2}°$. If the blade of the bevel is set at this angle, as at D, and the ends of the pieces of molding are cut to it, they will fit together properly.

EXAMPLE.—In Fig. 69, $G\,H\,E\,F$ and $H\,K\,D\,E$ represent two boards forming a center for supporting a brick arch during building; the shaded parts being cut off, leaving a curve, as $A\,L\,B$. What is the length of each board, and the angle of bevel $G\,H\,O$ where they meet? The radius $A\,O$ of the arch is 2 feet.

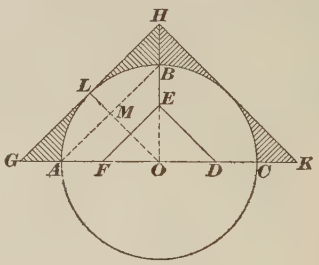

FIG. 69.

SOLUTION.—Draw $A\,B$ parallel to $G\,H$ and meeting $O\,G$ and $O\,H$, and also $O\,L$ perpendicular to $A\,B$; then $A\,M = M\,B$ (Art. **66**), and by similar triangles, $L\,G = L\,H$. $O\,L$ bisects the right angle $A\,O\,B$, hence, $A\,O\,M = \frac{1}{2}$ of $90°$, or $45°$. Then the other acute angle in the right triangle $O\,M\,A = 90° - 45° = 45°$ also. The angle $L\,G\,O$ is equal to angle $M\,A\,O$, or $45°$, and the right triangle $O\,L\,G$, having two angles equal, the opposite sides are equal also, and since $O\,L =$ the radius, or 2 feet, $L\,G$ is also 2 feet and $G\,H = 2 \times 2 = 4$ feet, the required length. Angle $O\,H\,G =$ angle $O\,G\,H = 45°$, the angle at which to set the bevel.

EXAMPLES FOR PRACTICE.

76. Solve the following:

1. What angle does the minute hand of a clock travel over in 5 minutes? Ans. $30°$.

2. The span of an arch is 7 feet, and the rise is 1 inch for each foot of span. What is the radius of the arch? Ans. 10 ft. $9\frac{1}{2}$ in.

3. The centering for an arch, Fig. 70, 8 feet in diameter is made of 3 pieces of equal length. Knowing that the side of a regular

inscribed hexagon is equal to the radius, what must be the length, as $A\,B$, of each piece?

Ans. 4.62 ft., or 4 ft. $7\frac{1}{2}$ in., nearly.

SUGGESTION.—Find the length of $O\,C$ in the right triangle $O\,C\,E$; then by similar triangles, find the length of $A\,B$.

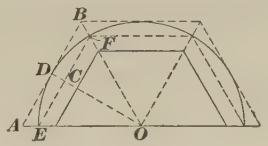

FIG. 70.

4. In the preceding example, what is (*a*) the bevel angle, and (*b*) the angle between any two pieces?

Ans. (*a*) 60°. (*b*) 120°.

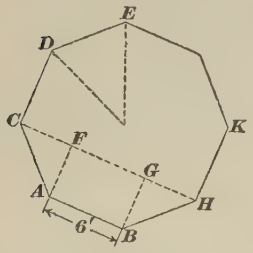

FIG. 71.

5. The flooring in a regular octagonal room, Fig. 71, is laid parallel with the side $A\,B$, which is 6 feet long. What are the angles of bevel for cutting the ends along $A\,C$ and $C\,D$?

Ans. 135° and 90°.

6. What is the length of the flooring between $C\,D$ and $H\,K$, Fig. 71, knowing that $C\,F$ and $A\,F$ are equal, and that angle $C\,F\,A$ is a right angle? (See Art. **48.**)

Ans. 14.48 ft., or 14 ft. $5\frac{3}{4}$ in., nearly.

MENSURATION.

INTRODUCTION.

77. Mensuration is that part of geometry which treats of the measurement of lines, surfaces, and solids.

78. The practical application of mensuration is the computation of lengths, areas of surfaces, or volumes of solids. The dimensions which furnish the data required are usually obtained from working drawings, or plans; therefore, before formally taking up the subject of mensuration, it is necessary to explain how drawings and plans are made, and how the dimensions are taken from them.

79. Working drawings are generally made to *scale;* that is, the lines on the drawing have a certain ratio to the corresponding dimensions of the full-size object which the drawing represents; this ratio is called the scale. For example, if the scale of a drawing is $\frac{1}{2}$ inch to 1 foot, it

means that each half inch on the drawing represents 1 foot on the object, and the drawing is $\frac{1}{2} \div 12 = \frac{1}{24}$ of full size.

80. To make drawings and to obtain measurements from them, divided rules called *scales* are used. These consist of strips or pieces of wood or metal 1 or 2 feet long, having the edges beveled, and upon which are engraved the graduations for the different scales, as 1 inch to 1 foot $\frac{1}{4}$ inch to 1 foot, etc. The marks are numbered in order from the end; the space next the end being subdivided into twelfths, representing inches, and in the larger scales, these spaces are again subdivided for fractions of an inch. There are usually two scales marked on each edge, and in one the divisions are twice as long as in the other, and are numbered from the other end.

To measure a line on a drawing, place the scale with the proper edge along the line, and move it until one of the foot marks is opposite one end of the line, as at *b*, Fig. 72, while

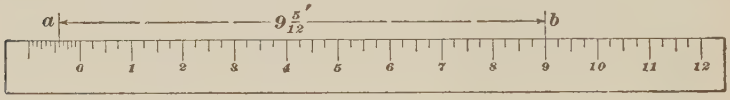

FIG. 72.

the other end of the line is at the *0* mark on the scale, or opposite one of the small graduations, as at *a*. Then read the number of large divisions from the *0* mark to the far end of the line, and also the number of small divisions from *0* to the near end of the line. Thus in Fig. 72, the line *a b* extends over 9 large spaces and 5 small ones, or $\frac{5}{12}$ of a large one, so that the length of the line represented by *a b* is 9 feet 5 inches.

81. Working drawings usually have the principal dimensions marked on them, as shown in Fig. 73, which represents a side and an end view of a cast-iron lintel. The length is 7 feet 6 inches, as indicated by the broken line *c d* extending between *e c* and *f d*, and drawn perpendicular to *e f* at the ends *e* and *f*. The arrowheads indicate the ends of the

7′ 6″ space. At *h* is shown that the height from *g* to *K* is 6 inches, being the distance between the parallel lines *m K* and *n g*. At *l* are shown two arrowheads pointed towards each other; these are used where the space is too small to

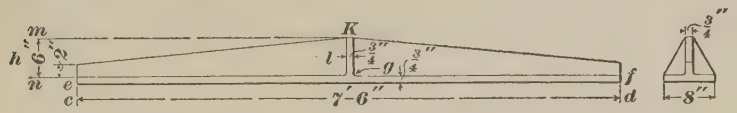

FIG. 73.

insert the arrowheads and figures in the ordinary manner. In this case it means that the thickness of the rib is $\frac{3}{4}$ inch, as shown by the nearby figures.

Having read or scaled all the necessary dimensions from the drawing, the proper formulas or rules may be applied to obtain the required length, area, or contents.

82. The following tables are useful in changing inches or fractions of an inch, to decimals of a foot or inch, and vice versa:

CONVERSION TABLES.

INCHES TO DECIMALS OF A FOOT.

Inches or Fractions.	Decimal of Foot.	Approximate Decimal.	Inches.	Decimal of Foot.	Approximate Decimal.
$\frac{1}{16}$	.0052	.005	3	.2500	.25
$\frac{1}{8}$	.0104	.010	4	.3333	.33
$\frac{1}{4}$	.0208	.020	5	.4167	.42
$\frac{3}{8}$	.0313	.030	6	.5000	.50
$\frac{1}{2}$	.0417	.040	7	.5833	.58
$\frac{5}{8}$	.0521	.050	8	.6667	.67
$\frac{3}{4}$	.0625	.060	9	.7500	.75
$\frac{7}{8}$	.0729	.070	10	.8333	.83
1	.0833	.080	11	.9167	.92
2	.1667	.170	12	1.0000	1.00

FRACTIONS OF AN INCH TO DECIMALS OF AN INCH.

Fraction.	Exact Decimal.	Approximate Decimal.	Fraction.	Exact Decimal.	Approximate Decimal.
$\frac{1}{32}$	.03125	.03	$\frac{9}{16}$	.5625	.56
$\frac{1}{16}$	.06250	.06	$\frac{5}{8}$	.6250	.63
$\frac{1}{8}$	.12500	.13	$\frac{11}{16}$	.6875	.69
$\frac{3}{16}$	.18750	.19	$\frac{3}{4}$	.7500	.75
$\frac{1}{4}$	.25000	.25	$\frac{13}{16}$	.8125	.81
$\frac{5}{16}$	.31250	.31	$\frac{7}{8}$	.8750	.88
$\frac{3}{8}$	.37500	.38	$\frac{15}{16}$	.9375	.94
$\frac{7}{16}$	.43750	.44	1	1.0000	1.00
$\frac{1}{2}$	.50000	.50			

EXAMPLE.— 7 ft. $8\frac{7}{8}$ in. = 7.00 ft. + .67 ft. + .07 ft. = 7.74 ft.
18.19 ft. = 18 ft. + .17 ft. + .02 ft. = 18 ft. $2\frac{1}{4}$ in.
$19\frac{9}{16}$ in. = 19.56 in., approximately.
13.26 in. = $13\frac{1}{4}$ in., approximately.

MENSURATION OF PLANE SURFACES.

83. The **area** of a surface is expressed by the number of unit squares it will contain.

84. A **unit square** is the square having the unit for its side. For example, if the unit is 1 inch, the unit square is the square whose sides measure 1 inch in length, and the area would be expressed by the number of square inches that the surface contains. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area would be the number of square feet that the surface contains, etc.

The square that measures 1 inch on a side is called a **square inch,** and the one that measures 1 foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft., or to $\square''$ and $\square'$.

THE TRIANGLE.

85. Rule.—*To find the area of a triangle, multiply the base by the altitude and divide the product by 2.*

Let b = base;
h = altitude;
A = area.

Then, $$A = \frac{b\,h}{2}.$$

EXAMPLE.—How many square feet of 1-inch boards will be needed for the 2 gables of a house having a half-pitch roof and width of 24 feet?

SOLUTION.—In half-pitch roofs the rise equals one-half the span; hence, $h = \frac{1}{2}$ of $24 = 12$ feet, and $b = 24$ feet. Applying the formula, $A = \frac{1}{2}\,b\,h = \frac{1}{2} \times 24 \times 12 = 144$ square feet for each gable, or $144 \times 2 = 288$ square feet of boards for the 2 gables. Ans.

86. If the triangle is a right-angled triangle, one of the short sides may be taken as the base, and the other short side as the altitude; hence, *the area of a right-angled triangle is equal to one-half the product of the two short sides.*

87. The area of any triangle may be found, when the length of each side is known, by means of the following formula, in which a, b, and c represent the lengths of the sides, s the half sum of the lengths, and A the area of the triangle:

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}, \text{ where } s = \frac{a+b+c}{2}.$$

EXAMPLE.—How many feet of 6-inch clapboards, laid $4\frac{1}{2}$ inches to the weather, will be required for the gable shown in Fig. 74?

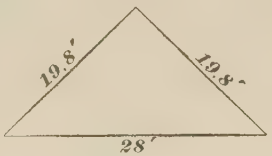

FIG. 74.

SOLUTION.—It is immaterial which side is called a, b, or c. Applying the formula, $s = \frac{a+b+c}{2} = \frac{28+19.8+19.8}{2} = 33.8$, the half sum; taking b and c as the short sides, $s-a = 33.8-28 = 5.8$ and $s-b$ and $s-c$ are each $33.8-19.8 = 14$. Now applying the formula,

$$A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)} = \sqrt{33.8 \times 5.8 \times 14 \times 14}$$
$$= 196+ \text{ square feet.}$$

A clapboard with $4\frac{1}{2}$-inch exposure and 1 foot long, has 54 square inches exposed; 1 square foot, then, will require $144 \div 54 = 2\frac{2}{3}$ linear feet of clapboards, and 196 square feet will require $196 \times 2\frac{2}{3} = 522\frac{2}{3}$ feet. Ans.

THE QUADRILATERAL.

88. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

FIG. 75.

89. A **rectangle** is a parallelogram whose angles are all right angles. See Fig. 75.

FIG. 76.

90. A **square** is a rectangle, all of whose sides are equal. See Fig. 76.

91. A **rhomboid** is a parallelogram whose opposite sides only are equal, and whose angles are not right angles. See Fig. 77.

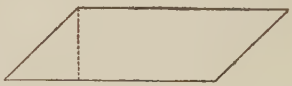

FIG. 77.

FIG. 78.

92. A **rhombus** is a parallelogram having equal sides, and whose angles are not right angles. See Fig. 78.

93. A **trapezoid** is a quadrilateral which has only two of its sides parallel. Fig. 79 is a trapezoid.

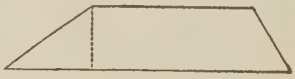

FIG. 79.

94. A **trapezium** is a quadrilateral having no two sides parallel. Fig. 80 is a trapezium.

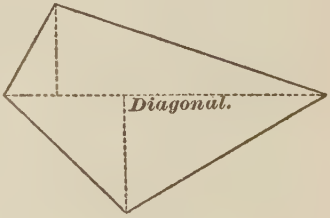

FIG. 80.

95. The **altitude** of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides.

96. A **diagonal** is a straight line drawn from the vertex of any angle of a quadrilateral to the vertex of the angle opposite; a diagonal divides the quadrilateral into two triangles. A diagonal divides a parallelogram into two *equal* and *similar* triangles.

97. **Rule.**—*To find the area of any parallelogram, multiply the base by the altitude.*

Let b = length of base;
h = altitude;
A = area.

Then, $A = b\,h$.

Example.—A lot is 4 rods wide and 8 rods long. (*a*) How many square feet does it contain? (*b*) What part of an acre?

Solution.—(*a*) Reducing rods to feet, 4 rods = 66 feet, and 8 rods = 132 feet. Either side may be taken as the base. Applying the formula,

$$A = b\,h = 66 \times 132 = 8{,}712 \text{ square feet.}$$ Ans.

(*b*) As there are 43,560 square feet in an acre, 8,712 square feet $= 8{,}712 \div 43{,}560 = .2 = \frac{1}{5}$ acre. Ans.

98. **Rule.**—*To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude of the trapezoid.*

Let a and b represent the lengths of the parallel sides, and h the altitude;

then, $$A = h\left(\frac{a+b}{2}\right).$$

Example.—How many square feet of floor space are there in a 5-story warehouse having the dimensions given in Fig. 81?

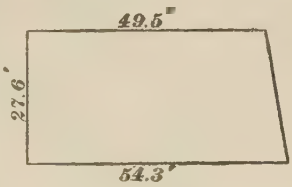

Fig. 81.

Solution.—As either side may be called a, let it be the shorter; then, applying the formula,

$$A = h\left(\frac{a+b}{2}\right) = 27.6\left(\frac{49.5+54.3}{2}\right)$$

$= 27.6 \times 51.9 = 1{,}432.44$ square feet.

Therefore, for 5 stories, there are 1,432.44 square feet $\times 5 = 7{,}162.2$ square feet of floor space. Ans.

POLYGONS.

99. Rule.—*To find the area of a regular polygon, divide the figure into isosceles triangles, compute the area of one triangle, and multiply by the number of triangles. The result will be the area of the polygon.*

Let n = number of sides of polygon;
l = length of one side;
h = perpendicular distance from the center to a side;
A = area of polygon.

Then, $$A = \frac{n\,l\,h}{2}.$$

EXAMPLE.—Find the floor surface of an octagonal room, the length of whose sides is 5 feet. The distance between parallel sides is 12 feet 1 inch nearly.

SOLUTION.—As the room is a regular octagon, $n = 8$, $l = 5$ feet, and $h = \frac{1}{2}$ of 12 feet 1 inch $= \frac{1}{2}$ of 12.08 feet $= 6.04$ feet. Applying the formula,

$$A = \frac{n\,l\,h}{2} = \frac{8 \times 5 \times 6.04}{2} = 120.8 \text{ square feet.} \quad \text{Ans.}$$

100. Rule.—*To find the area of an irregular polygon, or any figure bounded by straight lines, divide the figure into triangles, parallelograms, and trapezoids, and find the area of each. The sum of these partial areas will be the area of the figure.*

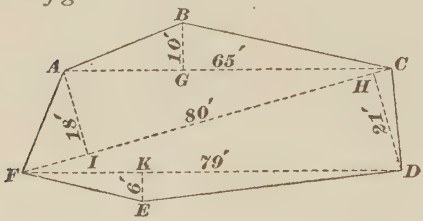

FIG. 82.

EXAMPLE.—A farmer fenced in a piece of land having the dimensions given in Fig. 82. How many square feet does it contain?

SOLUTION.—The distances marked on Fig. 82 being measured, the partial areas are found to be:

$$ABC = \frac{65 \times 10}{2} = 325 \text{ square feet;}$$

$$FAC = \frac{80 \times 18}{2} = 720 \text{ square feet;}$$

$$FCD = \frac{80 \times 21}{2} = 840 \text{ square feet;}$$

$$FDE = \frac{79 \times 6}{2} = 237 \text{ square feet.}$$

The area of the field is, therefore, the sum of these, or 2,122 square feet. Ans.

THE CIRCLE.

101. The ratio of the circumference of a circle to its diameter is an incommensurable number, and is usually denoted in technical books by the Greek letter π (pronounced *pi*). The approximate value of the ratio, correct to four decimal places, is 3.1416; hence, approximately $\pi = 3.1416$. For offhand calculations, the ratio is frequently taken as $3\frac{1}{7}$; that is, the circumference is $3\frac{1}{7}$ times the diameter.

102. **Rule.**—*To find the circumference of a circle, multiply the diameter by 3.1416.*

To find the diameter of a circle, divide the circumference by 3.1416.

103. Denoting the circumference by c, the diameter by d, and the radius by r,

$$c = \pi d = 2\pi r;$$

$$d = \frac{c}{\pi};$$

and

$$r = \frac{c}{2\pi}.$$

EXAMPLE.—A wheelwright wishes to cut a length of tire iron long enough to go around a $4\frac{1}{2}$-foot wagon wheel. How long must he cut it, allowing 4 inches for welding?

SOLUTION.—Here $d = 4\frac{1}{2}$ feet. Applying the formula,

$$c = \pi d = 3.1416 \times 4\tfrac{1}{2} = 14.14 \text{ feet.}$$

Adding the 4 inches = .33 foot, the length required is 14.47 feet. As this is only .03 foot = $\frac{3}{8}$ inch less than $14\frac{1}{2}$ feet, he would cut off $14\frac{1}{2}$ feet, and make the weld $4\frac{3}{8}$ inches.

104. As the circle is divided into 360 degrees, and the length of the circumference is $2\pi r$, the length of 1 degree is $\frac{2\pi r}{360}$; or, dividing both terms by 2π (= 6.2832), this reduces to $\frac{r}{57.3}$, very closely. If the angle is 57.3°, the length of the arc is equal to the radius; this angle is called a *radian*.

105. **Rule.**—*To find the length of an arc of a circle, multiply the number of degrees in the arc by the radius, and divide by 57.296 (or, sufficiently close, 57.3).*

Let l = length of arc;
n = number of degrees in arc;
r = radius of arc.

Then, $$l = \frac{r \times n}{57.3}.$$

If the angle contains degrees, minutes, and seconds, reduce them to degrees and decimals of a degree, thus:

$37° 30' 15'' = 37° + \frac{30}{60}° + \frac{15}{3600}° = 37° + .5° + .004° = 37.504°.$

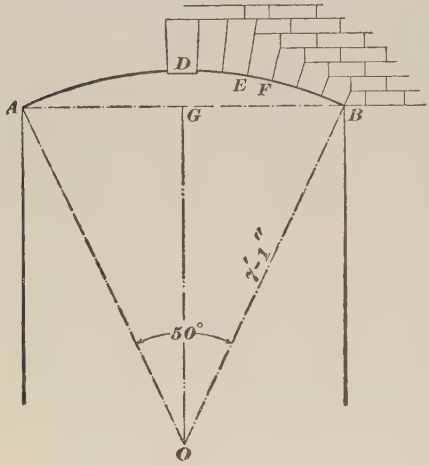

FIG. 83.

EXAMPLE.—In Fig. 83 is shown a segmental stone arch having a radius of 7 feet 1 inch = 85 inches. The angle $A\,O\,B$ is 50°. If there are 13 ring stones in the arch, what will be the width, as $E\,F$, of each stone?

SOLUTION. — Here $A\,D\,B = l$; $r = 85$ inches, and $n = 50°$. Applying the formula, $l = \frac{r\,n}{57.3} = \frac{85 \times 50}{57.3} = 74.17$ inches. Dividing by the number of stones, $\frac{74.17}{13} = 5.70$ inches $= 5\frac{11}{16}$ inches, nearly. Ans.

106. When the chord of the arc and the height of the segment (that is, $A\,B$ and $C\,D$, Fig. 84) are known, the length of the arc may be found by the following formula, in which

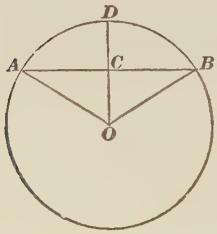

FIG. 84.

r = radius of arc;
l = length of the arc = length of $A\,D\,B$, Fig. 84;
c = length of the chord = length of $A\,B$;
h = height of segment = length of $C\,D$;

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}.$$

If the chord $A\,B$ and radius $O\,A$ are given, the height $C\,D$ of the segment may be found by the formula

$$h = r - \tfrac{1}{2}\sqrt{4\,r^2 - c^2}.$$

EXAMPLE.—If in Fig. 83 the span $A\,B$ is 6 feet, or 72 inches, and the rise $G\,D$ is 8 inches, what is the length of the intrados (arc $A\,D\,B$) of the arch?

SOLUTION.—Here the span $A\,B$ is the chord, and the rise is the height of the segment. Applying the formula,

$$l = \frac{4\sqrt{c^2 + 4\,h^2} - c}{3} = \frac{4\sqrt{72^2 + 4 \times 8^2} - 72}{3} = \frac{4 \times 73.76 - 72}{3} = 74.35$$

inches $= 74\frac{3}{8}$ inches, nearly. Ans.

107. When the quotient obtained by dividing the chord by the height is less than 4.8, that is, when $\frac{c}{h}$ is less than 4.8, the formula does not work well, the results not being sufficiently exact. In such a case, bisect the arc, and then apply the formula. Thus, in Fig. 85, suppose that $\frac{A\,B}{C\,D}$ is less than 4.8; then we should bisect $A\,C\,B$ by drawing $O\,C$ perpendicular to $A\,B$. We then find the length of the arc $A\,C$ and multiply the result by 2. But, in order to find the length of the arc $A\,C$, we must know the length of the radius $O\,A$ and the height of the segment included between the chord $A\,C$ and the arc $A\,C$. These may be found by means of the following formulas, in which

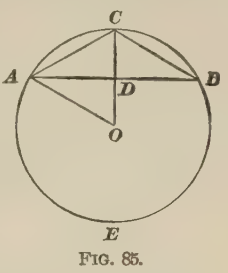

FIG. 85.

r = radius $O\,A$ of the arc;
C = chord $A\,B$ of the arc;
c = chord $A\,C$ of half the arc;
H = height $C\,D$ of the segment;
h = height of segment included between chord $A\,C$ and arc $A\,C$.

$$(a) \quad r = \frac{C^2 + 4\,H^2}{8\,H};$$

$$(b) \quad c = \tfrac{1}{2}\sqrt{C^2 + 4\,H^2};$$

$$(c) \quad h = r - \tfrac{1}{4}\sqrt{16\,r^2 - C^2 - 4\,H^2}.$$

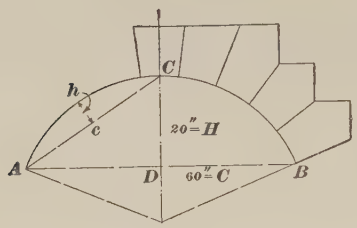

FIG. 86.

EXAMPLE.—The segmental arch in Fig. 86 has a span of 60 inches and a rise of 20 inches. What is the length of its intrados $A\,C\,B$?

SOLUTION.—Since $60 \div 20 = 3$, a number smaller than 4.8, we must find the length $A\,C$ of half the arc. Applying formula (a) to find the radius,

$$r = \frac{C^2+4H^2}{8H} = \frac{60^2+4\times 20^2}{8\times 20} = 32.5 \text{ inches.}$$

Now applying formula (b),

$$c = \tfrac{1}{2}\sqrt{C^2+4H^2} = \tfrac{1}{2}\sqrt{60^2+4\times 20^2} = 36.06 \text{ inches.}$$

Applying formula (c),

$$h = r-\tfrac{1}{4}\sqrt{16r^2-C^2-4H^2} = 32.5-\tfrac{1}{4}\sqrt{16\times 32.5^2-60^2-4\times 20^2}$$
$$= 5.46 \text{ inches.}$$

Applying the formula of Art. **106**,

$$l = \frac{4\sqrt{c^2+4h^2}-c}{3} = \frac{4\sqrt{36.06^2+4\times 5.46^2}-36.06}{3}$$
$$= 38.22 \text{ inches, length of arc } A\,C.$$

Therefore, $38.22\times 2 = 76.44$ inches, nearly = length of arc $A\,C\,B$. Ans.

108. Rule.—*To find the area of a circle, square the diameter and multiply by .7854; or, square the radius and multiply by 3.1416.*

Let d = diameter of circle;
r = radius of circle;
A = area of circle.

Then, $A = \frac{1}{4}\pi d^2 = .7854\,d^2$;
$A = \pi r^2 = 3.1416\,r^2$.

EXAMPLE.—What is the area of a circle whose radius is 14 inches?

SOLUTION.—Applying the formula,

$A = 3.1416\times 14^2 = 615.75$ square inches. Ans.

EXAMPLE.—The contract for a brick warehouse provides that all openings over 4 feet wide shall be deducted. What must be deducted for a semicircular arch 4 ft. 8 in. in diameter?

SOLUTION.—Here $d = 4$ feet 8 inches $= 4\frac{2}{3}$ feet. Applying the formula,

$A = .7854\,d^2 = .7854 \times (4\frac{2}{3})^2 = .7854 \times 21.78 = 17.11$ square feet.

For a half circle the area is $\frac{17.11}{2} = 8.55$ square feet, practically $8\frac{1}{2}$ square feet. Ans.

109. **Rule.**—*To find the diameter of a circle, the area being given, divide the area by .7854 and extract the square root of the quotient.*

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{4A}{\pi}}.$$

EXAMPLE.—To supply a certain quantity of water, it is necessary to have a pipe with an area of 28 square inches. What will be the diameter of the pipe?

SOLUTION.—Applying the formula,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{28}{.7854}} = \sqrt{35.65} = 5.97 \text{ inches.}$$

As this is almost 6 inches, a pipe of that diameter would be chosen. Ans.

110. **Rule.**—*To find the area of a flat circular ring, subtract the area of the smaller circle from that of the larger.*

Let d = the longer diameter;
d_1 = the shorter diameter;
A = area of ring.

Then, $A = .7854\,d^2 - 7854\,d_1^2 = .7854(d^2 - d_1^2)$.

EXAMPLE.—What is the sectional area of a brick stack whose external and internal diameters are 6 feet 6 inches, and 4 feet respectively?

SOLUTION.—Here $d = 6.5$ feet, and $d_1 = 4$ feet. Applying the formula,

$A = .7854(6.5^2 - 4^2) = .7854 \times 26.25 = 20.62$ square feet. Ans.

If one diameter and the area of the ring are known, the other diameter may be found by adding to or subtracting from the area of the given circle that of the ring, and finding the diameter corresponding to the resulting area.

111. Rule.—*To find the area of a sector of a circle, divide the number of degrees in the arc of a sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

Let n = number of degrees in arc;
A = area of circle;
r = radius of circle;
a = area of sector.

Then, $$a = \frac{n\,A}{360} = .0087267\,n\,r^2.$$

Example.—A circular window 4 feet in diameter is divided by radial ribs into 12 equal sections. What is the area of each space?

Solution.—Here $n = \frac{360^\circ}{12} = 30^\circ$. Applying the formula,

$$a = \frac{n\,A}{360} = \frac{30 \times .7854 \times 4^2}{360} = 1.05 \text{ square feet.} \quad \text{Ans.}$$

Or, $a = .0087267\,n\,r^2 = .0087267 \times 30 \times 4 = 1.05$ square feet. Ans.

112. The area may also be found as follows:

Rule.—*To find the area of a sector, multiply one-half of the length of the arc by the radius.*

Let l = length of the arc;
r = radius of the arc;
a = area of sector.

Then, $$a = \tfrac{1}{2}\,l\,r.$$

Example.—In the sector $O\,A\,D\,B$, Fig. 84, the radius ($O\,A = O\,D$) of the arc is 6 inches, and the length of the chord $A\,B$ is 7 inches; what is the area of the sector?

Solution.—In order to find the area, we must first find the length of the arc, by applying the formula given in Art. **106**; but before applying this formula, we must find the height $C\,D$ of the segment. Since $AC = \frac{1}{2}\,AB = \frac{1}{2} \times 7 = 3.5$, $OC = \sqrt{\overline{OA}^2 - \overline{AC}^2} = \sqrt{6^2 - 3.5^2} = 4.87$ inches. Then, $CD = OD - OC = 6 - 4.87 = 1.13$ inches.

Applying the formula of Art. **106**,

$$l = \frac{4\sqrt{7^2 + 4 \times 1.13^2} - 7}{3} = 7.48 \text{ inches.}$$

Now applying the formula given above,

$$a = \tfrac{1}{2}\,l\,r = \tfrac{1}{2} \times 7.48 \times 6 = 22.44 \text{ square inches.} \quad \text{Ans.}$$

113. **Rule.**—*To find the area of a segment of a circle, find the area of the sector of which the segment is a part, and from this area subtract the area of the triangle formed by drawing radii to the extremities of the chord of the segment. The result is the area of the segment.*

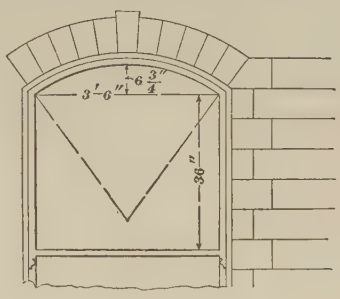

FIG. 87.

EXAMPLE.—What is the area of the upper pane of glass in the window shown in Fig. 87?

SOLUTION.—The radius is found by applying the formula

$$r = \frac{c^2 + 4h^2}{8h};$$

$c = 3$ feet 6 inches $= 42$ inches, and $h = 6\frac{3}{4}$ inches $= 6.75$ inches. Substituting,

$$r = \frac{42^2 + 4 \times 6.75^2}{8 \times 6.75} = 36 \text{ inches.}$$

The length of the arc is

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3} = \frac{4\sqrt{42^2 + 4 \times 6.75^2} - 42}{3} = 44.83 \text{ inches.}$$

The area of the sector is

$$a = \frac{lr}{2} = \frac{44.83 \times 36}{2} = 806.94 \text{ square inches.}$$

The altitude of the triangle is 36 inches $- 6\frac{3}{4}$ inches $= 29\frac{1}{4}$ inches, and its area is $\frac{42 \times 29\frac{1}{4}}{2} = 614.25$ square inches. The area of the segment is, therefore, $806.94 - 614.25 = 192.69$ square inches. The area of the rectangular portion of the glass is $42 \times 36 = 1{,}512$ square inches; adding to this 192.69, there results 1,704.69 square inches. Ans.

THE ELLIPSE.

114. An ellipse is a plane figure bounded by a curved line, to any point of which the sum of the distances from two fixed points within, called the foci, is equal to the sum of the distances from the foci to any other point on the curve.

In Fig. 88 let A and B be the foci, and let C and D be any two points on the perimeter. Then, according to the above definition, $AC + CB = AD + DB$, and both these sums are also equal to the major axis EF. The long diameter EF is called the major axis; the short diameter GD, the minor axis. The foci may be located from D or G as a center, and radius $DA = DB = \frac{1}{2}EF$.

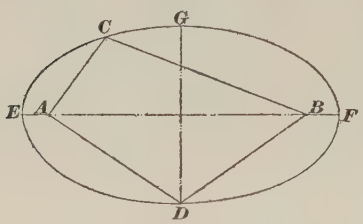

FIG. 88.

115. There is no exact method of finding the perimeter of an ellipse; but the following is close enough for most cases:

Rule.—*Multiply the major axis by 1.82, and the minor axis by 1.315. The sum of the results will be the perimeter.*

Let D = major diameter;
d = minor diameter;
C = perimeter, or circumference.

Then, $$C = 1.82\,D + 1.315\,d.$$

116. **Rule.**—*To find the exact area of an ellipse, multiply the product of its two diameters by .7854.*

$$A = \tfrac{1}{4}\pi\,d\,D = .7854\,d\,D,$$

where A represents the area, and D and d the two diameters.

EXAMPLE.—The soffit (under surface) of the semi-elliptic stone arch shown in Fig. 89 is to be carved at a cost of $3.50 per square foot. If the arch is 15 inches wide, what will the carving cost?

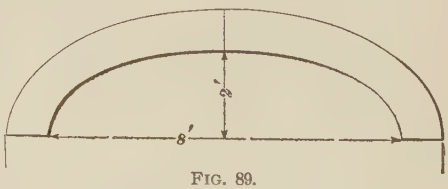

FIG. 89.

SOLUTION.—Here $D = 8$ feet, and $d = 2 \times 2 = 4$ feet; hence, applying the formula,

$$C = 1.82\,D + 1.315\,d = 1.82 \times 8 + 1.315 \times 4 = 19.82 \text{ feet.}$$

For one-half the ellipse, the length is $\frac{19.82}{2} = 9.91$ feet. The area is 9.91×1.25 (= 15 in.) = 12.39 square feet. Therefore, the cost of carving is $12.39 \times \$3.50 = \$43.36$. Ans.

EXAMPLE.—What is the area of the elliptical top of a table whose long and short diameters are 4 and 3 feet, respectively?

SOLUTION.—Here $D = 4$ feet and $d = 3$ feet. Applying the formula,

$$A = .7854\, D\, d = .7854 \times 4 \times 3 = 9.42 \text{ square feet.}$$ Ans.

AREA OF ANY PLANE FIGURE.

117. Rule.—*To find the area of any plane figure, divide it into triangles, quadrilaterals, circles, or parts of circles, and ellipses, find the area of each part separately and add the partial areas.*

EXAMPLE.—Find the sectional area of the half-arch and abutment shown in Fig. 90.

SOLUTION.—Divide the section into parts, as $a\,b\,i\,h$, $e\,i\,g\,d$, and $b\,g\,c$. Find $f\,h = f\,o - h\,o = 16 - \sqrt{16^2 - 8^2} = 2.14$ feet; then,

area $a\,b\,i\,h = 12 \times 5.14 = 61.68$ sq. ft.;

area $e\,i\,g\,d = 8 \times 4 = 32$ sq. ft.;

area $b\,g\,c = \frac{3 \times 13.14}{2} = 19.71$ sq. ft.

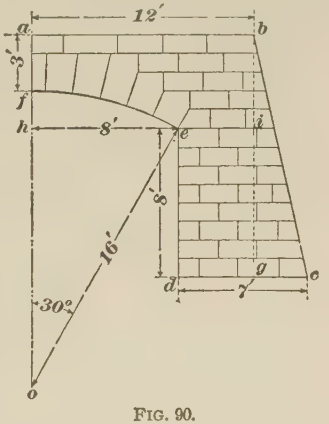

FIG. 90.

Adding these partial areas, the sum is 113.39 square feet. The area of sector $f\,o\,e = .0087267\, n r^2 = .0087267 \times 30 \times 16^2 = 67.02$ square feet. The area of triangle $h\,e\,o = \frac{1}{2}(h\,o \times e\,h)$; $h\,o = 16 - 2.14 = 13.86$ feet; area $= \frac{13.86 \times 8}{2} = 55.44$ square feet. The area of segment $f\,e\,h = 67.02 - 55.44 = 11.58$ square feet. Deducting 11.58 square feet from 113.39 square feet, the net area = 101.81 square feet. Ans.

AREAS OF IRREGULAR FIGURES.

118. If the figure is so irregular in shape that the ordinary rules cannot be easily applied, the area may be obtained as follows: Let Fig. 91 represent a drawing of such an area. Suppose it to be divided by equidistant parallel lines into strips, as $a\,b$, $b\,c$, etc.; then each of these small areas may be considered a trapezoid, and the whole area is the sum of the partial areas.

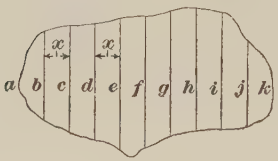

FIG. 91.

From these considerations we obtain the following rule:

119. **Rule.**—*To find the area of any irregular figure, divide the figure into any number of strips by equidistant parallel lines. Measure the length of the lines; add together one-half the lengths of the end lines and the lengths of the remaining lines, and multiply this sum by the distance between the lines.*

Let a = length of one end line;
k = length of other end line;
$b, c,$ etc. = lengths of intermediate lines;
x = distance between parallels;
A = area.

Then, $$A = \left(\frac{a+k}{2} + b + c + \text{etc.} + j\right)x.$$

The shorter the distance x is, the more accurate will be the result.

EXAMPLE.—Fig. 92 is a map of a small island whose area is to be determined. The parallel lines are 10 feet apart, and are of the lengths marked thereon. Required, the area of the island.

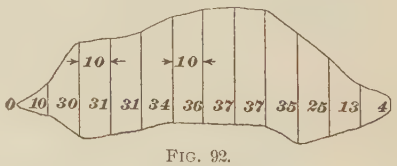

FIG. 92.

SOLUTION.—The extreme left end of the island is a point; hence, its length, or a, is 0; k is 4 feet.

Applying the formula, $A = \left(\frac{a+k}{2} + b + \text{etc.}\right)x$

$$= \left(\frac{0+4}{2} + 10 + 30 + 31 + 31 + 34 + 36 + 37 + 37 + 35 + 25 + 13\right) \times 10$$

$$= 321 \times 10 = 3{,}210 \text{ sq. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

(*See note at bottom of page 12.*)

120. Solve the following examples:

1. (*a*) How many squares (100 sq. ft.) of shingling, and (*b*) how many bundles of shingles, 250 to the bundle, each shingle being 6 inches wide and laid 5 inches to the weather, will be required for the gable shown in Fig. 93, adding 5 per cent. for waste?

Ans. { (*a*) $2\frac{1}{4}$ squares. (*b*) $4\frac{1}{2}+$ bundles.

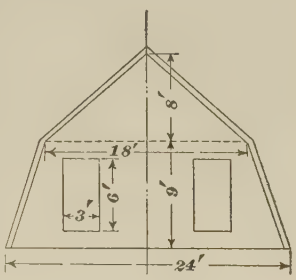

FIG. 93.

SUGGESTION.—Gable consists of trapezoid and triangle. Find exposed area of each shingle and number required per square of 14,400 square inches.

2. What is the cost of plastering the wall and ceiling of a semicircular alcove, 6 feet in diameter and 8 feet high, at 35 cents per square yard? Ans. $3.48.

SUGGESTION.—Total area = area of semicircle + (length of semicircumference × height of wall).

3. What is the length of the intrados *A B E C D* of the 3-centered arch shown in Fig. 94? Ans. 16.76 ft.

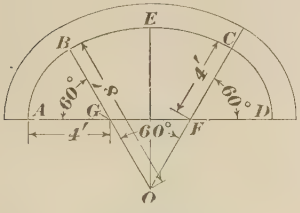

FIG. 94.

SUGGESTION.—Find lengths of arcs *B E C*, *A B*, and *C D* separately. *A B* = *C D*.

4. The space above the line *A D*, in Fig. 94, is to be covered with grille-work, costing $1.50 per square foot. What will be the total cost? Ans. $65.02.

SUGGESTION.—Area required = area of three sectors less the area of triangle *G O F*. *G F* = *G O* = *F O*, because angles are all equal; then lengths of three sides are known, to find the area of triangle.

5. How many laths 4 feet long and $1\frac{1}{2}$ inches wide, spaced $\frac{1}{4}$ inch apart, transversely, will be required for the walls of a room 14 ft. 6 in.

$\times$ 10 ft. 9 in. $\times$ 8 ft. high, deducting 2 doors 36 in. $\times$ 7 ft. and 2 windows 2 ft. 6 in. $\times$ 5 ft. 6 in. ? Add 5% for waste. Ans. 601 laths.

SUGGESTION.—Number of lath per square foot = 144 $\div$ [(width of 1 lath + 1 space) $\times$ length of lath in inches].

6. A hollow circular cast-iron column, having an external diameter of 8 inches, is to carry a load of 47,120 pounds, and must not be loaded more than 4,000 pounds per square inch. What will be the thickness of the metal ring? Ans. $\frac{1}{2}$ in.

SUGGESTION.—Area of ring = total load $\div$ load per square inch. Area of smaller circle = area of 8-inch circle − area of ring; from this find smaller diameter.

7. If brick masonry will safely carry 10 tons per square foot, what must be the size of the square base of the column given in the previous example? (Tons of 2,000 lb.) Ans. 1.54 ft., or $18\frac{1}{2}$ in., square.

SUGGESTION.—Side of square = square root of area required.

FIG. 95.

8. A bar of iron 1 yard long and 1 inch square weighs 10 pounds. What will be the weight per yard of the I beam shown in Fig. 95? Disregard curved edges and corners. All work in fractions.

Ans. $41\frac{2}{3}$ lb. per yd.

SUGGESTION.—Area required = areas of 3 rectangles + 4 triangles. Length $i\ e$ = 6 inches − $(\frac{5}{16} \times 2)$. Express results in 256ths of a square inch, and reduce the sum.

9. What is the area of the cross-section of the dam shown in Fig. 96? $K\ C$ = $D\ K$, and $O\ C$ and $O\ D$ are perpendicular, respectively, to $K\ C$ and $D\ K$.

Ans. 138.93 sq. ft.

SUGGESTION.—Divide area into trapezoids, rectangles, etc. Area $D\ M\ C\ K$ = areas of equal right triangles $D\ O\ K$ and $K\ O\ C$ less area of sector. The angle $D\ O\ C$ being 63° 26′ = 63.43°, find length of arc $D\ C$, and area of sector. Area $D\ M\ C\ K$ = 7.53 sq. ft.

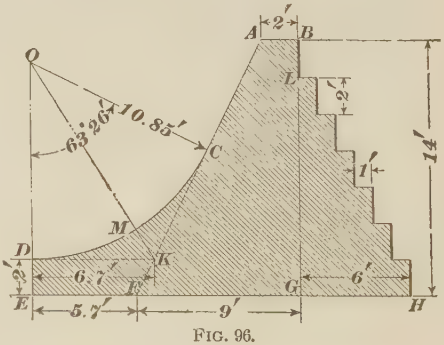

FIG. 96.

10. The segment $A\ B\ C$ of the window shown in Fig. 97 is to be filled with ornamental glass. What is the cost at \$6 per square foot? Ans. \$5.64.

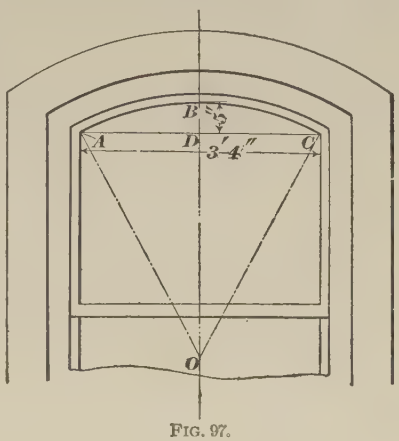

FIG. 97.

SUGGESTION.—Area = area of sector − area of triangle. Find radius $A\ O$ and arc $A\ B\ C$ by proper formulas; OD = radius − 5 inches.

11. (*a*) What is the length of sash for an elliptical window whose axes are 8 feet and 4 feet 8 inches? (*b*) If the sash is 4 inches wide, what is the exposed area of the glass?

Ans. { (*a*) 20.70 ft.
(*b*) 23.03 sq. ft.

SUGGESTION.—The axes of the glass are 4 inches × 2 = 8 inches less than those of the sash.

12. In an octagonal room having equal sides the distance between the parallel sides is 12 feet and the distance between opposite corners is 13 feet. How many feet B. M. of 1-inch white-pine flooring will be required? Add 5% for waste and poor stuff. Ans. 126 ft. B. M.

SUGGESTION.—Length of side = 2 times base of triangle. Area = 8 triangles.

13. Pressed brick are estimated at 7 per square foot of external area. How many will be required for a building 20 ft. × 40 ft. and 20 feet high; with 2 chimneys 20 in. × 28 in. and 8 feet high; and 1 chimney 20 in. × 28 in. and 12 feet high? Deduct 6 window openings, 3 ft. × 6 ft. 6 in.; 1 double window, 2 openings, each 2 ft. 8 in. × 9 ft.; 1 front door, 4 ft. 8 in. × 9 ft.; 1 rear door, 3 ft. × 9 ft.; 6 window openings, 3 ft. × 6 ft.; and 1 double window, 2 openings, 2 ft. 8 in. × 6 ft.

Ans. 15,750 brick.

14. (*a*) How many squares (100 square feet) of slating are there on a house 25 ft. × 40 ft. with half-pitch roof, supposing the roof to extend beyond the walls 1 foot (along the roof) at eaves and ends? (*b*) How many slates, 14 inches wide and exposed 8½ inches, will be required?

Ans. { (*a*) 15.69 squares.
(*b*) 1,899 slates.

SUGGESTION.—Rise of roof = one-half width. Find exposed area of each slate and number required per square of 14,400 square inches.

THE MENSURATION OF SOLIDS.

DEFINITIONS.

121. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their intersections are called **edges.**

122. The **entire surface** of a solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

123. The **volume** of a solid is expressed by the number of times it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

THE PRISM AND CYLINDER.

124. A **prism** is a solid whose ends are equal parallel polygons and whose sides are parallelograms.

Prisms take their names from their bases. Thus, a triangular prism is one whose bases are triangles; a pentagonal prism is one whose bases are pentagons, etc.

FIG. 98.

125. A **parallelopiped** is a prism whose bases (ends) are parallelograms. Fig. 98.

126. A **cube** is a parallelopiped whose faces and ends are squares. Fig. 99.

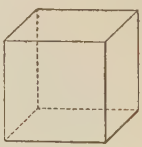

FIG. 99.

127. The cube whose edges are equal to the unit of length is taken as the unit of volume when finding the volume of a solid.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges measure 1 inch, or 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

128. A **cylinder** is a solid whose ends are equal and similar *curved* figures. A *circular cylinder* is one any section of which, perpendicular to the axis, is a circle. See Fig. 100. Unless otherwise expressed, the word *cylinder* always means a *circular* cylinder.

FIG. 100.

129. A *right* prism or *right* cylinder is one whose center line (axis) is perpendicular to its base.

130. The *altitude* of a prism or cylinder is the perpendicular distance between its two ends.

131. **Rule.**—*To find the area of the convex surface of any right prism, or right cylinder, multiply the perimeter of the base by the altitude.*

Let p = perimeter of base;
h = altitude;
S = convex surface.

Then, $S = p\,h$.

To find the entire area, add the areas of the two ends to the convex areas.

EXAMPLE.—How many feet, board measure, of 1-inch sheathing are needed for a house 20 ft. $\times$ 28 ft. $\times$ 18 ft. high, including gables under a half-pitch roof, making no allowance for openings?

SOLUTION.—The perimeter p of the house $= 20 \times 2 + 28 \times 2 = 96$ feet; $h = 18$ feet; hence, $S = 96 \times 18 = 1{,}728$ square feet. Adding 2 gables, 20 feet base, and (since the roof is half pitch) 10 feet high $= 200$ square feet, the total is 1,928 square feet. As the sheathing is 1 inch thick the feet B. M. is also 1,928. Ans.

EXAMPLE.—A circular iron chimney, 7 feet in diameter and 85 feet high, is to be covered outside with a preservative paint. What will it cost to paint at 20 cents per square yard?

SOLUTION.—The perimeter p of the base is $\pi d = 21.99$ feet, and $h = 85$ feet. The surface of the chimney is $21.99 \times 85 = 1{,}869.2$ square feet, or 207.7 square yards. The cost of painting at 20 cents per square yard is, therefore, $207.7 \times \$.20 = \$41.54$. Ans.

EXAMPLE.—How many square feet of zinc will be required to line a refrigerator having interior dimensions of 4 ft. $\times$ 6 ft. $\times$ 8 ft. high?

SOLUTION.—The area of the sides $= (4 \times 2 + 6 \times 2) \times 8 = 160$ square feet. Adding for floor and ceiling $(4 \times 6) \times 2 = 48$ square feet, the total is 208 square feet. Ans.

EXAMPLE.—What will be the cost, at 35 cents per square yard, of plastering the walls and ceiling of an octagonal room, having sides 5 feet long and 11 feet high, the distance between the parallel sides being 12 feet?

SOLUTION.—The area of the sides is $8(5 \times 11) = 440$ square feet, the area of the ceiling is $\frac{8 \times 5 \times 6}{2} = 120$ square feet; the total is 560 square feet, or 62.22 square yards. At 35 cents per square yard, the cost is \$21.78. Ans.

132. Rule.—*To find the volume of a right prism, or cylinder, multiply the area of the base by the altitude.*

Let a = area of base;
h = altitude;
V = volume.

Then, $$V = a\,h.$$

If the given prism is a cube, the three dimensions are all equal, and the volume is equal to the cube of one of the edges.

If the volume and area are given, the altitude $= \frac{V}{a}$. If the cylinder or prism is hollow, the volume is equal to the area of the ring multiplied by the altitude.

EXAMPLE.—If brickwork averages 21 brick per cubic foot, how many brick are there in a pier 18 inches square and 6 feet high?

SOLUTION.—The sectional area or a is $1.5 \times 1.5 = 2.25$ square feet; $h = 6$ feet; hence, $V = a\,h = 2.25 \times 6 = 13.5$ cubic feet; at 21 brick per cubic foot, the number of bricks in the pier is $13.5 \times 21 = 283.5$. Ans.

EXAMPLE.—If cast iron weighs .26 pound per cubic inch, what length of sash weight will be necessary to balance a window weighing 8 pounds, the diameter of the cylindrical weight being $1\frac{1}{2}$ inches?

SOLUTION.—As there are 2 weights, the weight of each is 4 pounds. The number of cubic inches required, or V, is $4 \div .26 = 15.38$. The area a of a $1\frac{1}{2}$-inch circle is 1.77 square inches; hence, h, the length, is $\frac{V}{a} = \frac{15.38}{1.77} = 8.69$ in., practically $8\frac{3}{4}$ in. Ans.

THE PYRAMID AND CONE.

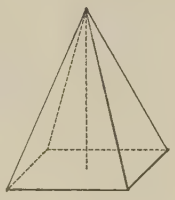

FIG. 101.

133. A **pyramid** is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.** See Fig. 101.

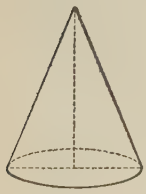

FIG. 102.

134. A **cone** is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.** See Fig. 102.

135. A **right pyramid** or **cone** is one whose axis is perpendicular to the base.

136. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

137. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

138. **Rule.**—*To find the convex area of a right pyramid or cone, multiply the perimeter of the base by one-half of the slant height.*

Let p = perimeter of base;
s = slant height;
A = convex area.

Then, $$A = \frac{ps}{2}.$$

EXAMPLE.—An octagonal church steeple is 60 feet high. The outer radius of the base (see Fig. 103) is 6 feet, and the length of one of the sides of the base is 4.6 feet. Compute the convex surface.

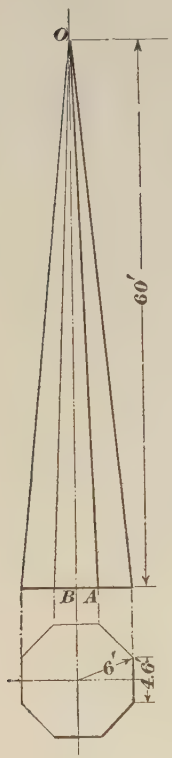

FIG. 103.

SOLUTION.—To find the slant height, the length OA along the hips must first be found. This is the hypotenuse of a right-angled triangle of which the altitude is 60 feet, the height of the tower, and the base is the radius, 6 feet. Hence $OA = \sqrt{60^2 + 6^2} = 60.3$. Now, in the triangle OAB, $OA = 60.3$ feet and $AB = \frac{4.6}{2} = 2.3$ feet. The slant height is, therefore, $OB = \sqrt{60.3^2 - 2.3^2} = 60.3$ feet, nearly. The perimeter of the base is $4.6 \times 8 = 36.8$ feet. Applying the formula, $A = \frac{ps}{2} = \frac{36.8 \times 60.3}{2} = 1{,}109.5$ sq. ft. Ans.

139. Rule.—*To find the volume of a pyramid or cone, multiply the area of the base by one-third of the altitude.*

Let a = area of base;
h = altitude;
V = volume.

Then, $$V = \frac{ah}{3}.$$

EXAMPLE.—The granite capstone of a monument is a square pyramid having a base 4 feet square and an altitude of 6 feet. If granite weighs 170 pounds per cubic foot, what will be the freight charges on the piece at 20 cents per 100 pounds?

SOLUTION.—The area a of the base is $4 \times 4 = 16$ square feet, and the altitude h is 6 feet. Applying the formula, $V = \frac{ah}{3} = \frac{16 \times 6}{3} = 32$ cubic feet; at 170 pounds per cubic foot, the weight is $32 \times 170 = 5{,}440$ pounds $= 54.4$ hundredweight. The freight is therefore $54.4 \times \$.20 = \$10.88$. Ans.

EXAMPLE.—If in the preceding example the capstone were conical, having a base 4 feet in diameter and a height of 6 feet, how much less would it weigh than the pyramidal stone?

SOLUTION.—The area of the base is $.7854 d^2 = 12.57$ square feet. Using the formula, the volume is $\frac{12.57 \times 6}{3} = 25.14$ cubic feet, and the weight at 170 pounds per cubic foot is 4,274 pounds, nearly. As the pyramidal capstone weighs 5,440 pounds, the difference in weight is $5{,}440 - 4{,}274 = 1{,}166$ lb. Ans.

THE FRUSTUM OF A PYRAMID OR CONE.

140. If a pyramid or cone is cut by a plane parallel to the base, so as to form two parts, the lower part is called the frustum of the pyramid or cone. See Figs. 104 and 105.

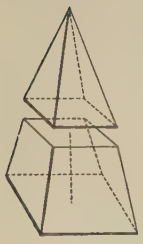

FIG. 104.

The upper end of the frustum of a pyramid or cone is called the upper base, and the lower end the lower base. The altitude of a frustum is the perpendicular distance between the bases.

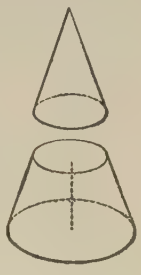

FIG. 105.

141. **Rule.**—*To find the convex area of a frustum of a right pyramid or right cone, multiply one-half the sum of the perimeters of the bases by the slant height.*

Let P = perimeter of lower base;
p = perimeter of upper base;
s = slant height;
A = convex area.

Then, $$A = \frac{(P+p)s}{2}.$$

To find the entire area, add to the convex area the areas of the two bases.

EXAMPLE.—A square mansard-tower roof has the dimensions shown in Fig. 106. To compute the slate required to cover the tower the convex area is required. What is this convex area?

SOLUTION.—The tower has the form of a frustum of a pyramid. 6 ft. 6 in. = $6\frac{1}{2}$ feet and 3 ft. 6 in. = $3\frac{1}{2}$ feet. The perimeter of the lower base is $4 \times 6\frac{1}{2} = 26$ feet, and that of the upper base is $4 \times 3\frac{1}{2} = 14$ feet. In the triangle $A\,B\,C$, $A\,C = 6$ feet, and $B\,C = \frac{1}{2}(6\frac{1}{2} - 3\frac{1}{2}) = 1\frac{1}{2}$ feet. Hence, the hypotenuse $A\,B$, which is the slant height, is $\sqrt{\overline{B\,C}^2 + \overline{A\,C}^2} = \sqrt{1\frac{1}{2}^2 + 6^2} = 6.18$ feet.

Applying the formula, $A = \frac{(P+p)s}{2}$

$= \frac{(26+14) \times 6.18}{2} = 123.6$ sq. ft. Ans.

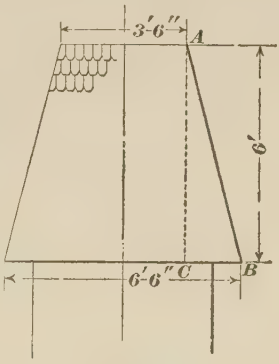

FIG. 106.

142. Rule.—*To find the volume of the frustum of a pyramid or cone, add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.*

Let A = area of lower base;
a = area of upper base;
h = altitude;
V = volume.

Then, $$V = \frac{h}{3}(A + a + \sqrt{A \times a}).$$

EXAMPLE.—A marble monument is $2\frac{1}{2}$ feet square at the base, 1 foot square at the top, and is 16 feet high. If marble weighs 160 pounds per cubic foot, what is the weight of the stone?

SOLUTION.—The area of the lower base is $2\frac{1}{2} \times 2\frac{1}{2} = 6.25$ square feet; that of the upper base is 1 square foot. Applying the formula,

$$V = \frac{h}{3}(A + a + \sqrt{A \times a}) = \frac{16}{3}(6.25 + 1 + \sqrt{6.25 \times 1}) = 51.97 \text{ cu. ft.};$$

at 160 pounds per cubic foot, the weight is $51.97 \times 160 = 8{,}315$ lb.

THE WEDGE.

143. A wedge is a solid having plane surfaces, of which the base is a parallelogram, the ends are triangles, and the sides are quadrilaterals meeting in a line parallel to the sides of the base. See Fig. 107.

FIG. 107.

144. Rule.—*To find the volume of a wedge, multiply together the width of the base, the perpendicular distance from the base to the edge, and the sum of the lengths of the three parallel edges; divide the product by 6.*

Let w = width of base;
h = perpendicular distance from base to edge;
s = sum of lengths of the three parallel edges;
V = volume.

Then, $$V = \frac{w\,h\,s}{6}.$$

If the base is a rectangle, and the triangular ends are parallel, the wedge becomes a triangular prism, and the rule for prisms may be used.

EXAMPLE.—Steel weighs .28 pound per cubic inch. How heavy is a steel wedge 8 inches long, and $1\frac{1}{2}$ in. $\times$ 3 in. at the head?

SOLUTION.—Here w is $1\frac{1}{2}$ inches, h is 8 inches, s is $3+3+3=9$ inches. Then $V = \frac{1.5 \times 8 \times 9}{6} = 18$ cubic inches. Or, as this wedge is a prism, the volume = area of end $\times$ length of edge; area of triangle $= \frac{1.5 \times 8}{2} = 6$ square inches; multiplying by the length of the edge, 3 inches, the volume is 18 cubic inches. The weight at .28 pound per cubic inch is $.28 \times 18 = 5.04$ lb. Ans.

THE PRISMOIDAL FORMULA.

145. If any solid be sliced in pieces whose adjacent surfaces are flat, any piece is called a **plane section** of the solid.

Plane sections are divided into three classes: Longitudinal sections, cross-sections, and right sections. A **longitudinal section** is any plane section taken lengthwise through the solid. Any other plane section is called a **cross-section.** If the surface exposed by taking a plane section of a solid is perpendicular to the center line of the solid, the section is called a **right section.** The surface exposed by any longitudinal section of a cylinder is a rectangle. The surface exposed by a right section of a cube is a square; of a cylinder or cone, a circle; an oblique cross-section of a cylinder is an ellipse.

146. A **prismoid** is a solid having for its ends two *parallel* plane surfaces which are connected by plane triangular or quadrilateral faces.

Thus, the solid shown in Fig. 108 is a prismoid. The parallel ends are the pentagon *A B C D E* and the quadrilateral *F G H I*, and these ends are connected by the triangular face *B C H* and the four quadrilateral faces *A B H I*, *A E F I*, *D E F G*, and *C D G H*.

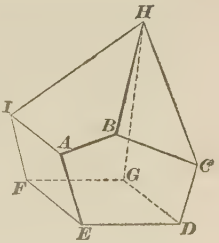

FIG. 108.

147. Rule.—*To find the volume of a prismoid, add together the areas of the two parallel ends, and four times the*

area of a parallel section midway between them; multiply the sum by one-sixth of the perpendicular distance between the parallel ends.

Let A = area of one end;
a = area of other end;
M = area of middle section;
h = distance between ends;
V = volume.

Then, $$V = \frac{h}{6}(A + a + 4M).$$

The area of the middle section is not in general a mean between the end areas, but the lengths of its sides are means between the corresponding lengths on the ends. This formula, called the *prismoidal formula*, is of very extended application and may be used to calculate the volume of a prism, cylinder, pyramid, cone, frustum of pyramid and cone, wedge, sphere, and segments of spheres, in addition to the irregularly shaped bodies to which it is usually applied. For a pyramid, cone, and wedge, the upper area is 0; for a sphere, the end areas are both 0; for a cylinder and prism, the areas are all equal.

EXAMPLE.—What is the volume in cubic yards of a masonry pier 30 feet high, the upper and lower rectangular bases being, respectively, 7 ft. $\times$ 24 ft. and 11 ft. $\times$ 30 ft.?

SOLUTION.—The area of the upper base is $7 \times 24 = 168$ square feet. The area of the lower base is $11 \times 30 = 330$ square feet. The dimensions of the middle section are the means of the corresponding dimensions of the bases: therefore, the width is $\frac{24+30}{2} = 27$ feet, and the thickness is $\frac{7+11}{2} = 9$ feet. The area of the middle section is $27 \times 9 = 243$ square feet. Applying the prismoidal formula, the volume is $V = \frac{30}{6}(330 + 168 + 4 \times 243) = 7{,}350$ cubic feet $= \frac{7{,}350}{27}$ cubic yards $= 272\frac{2}{3}$ cu. yd. Ans.

148. The prismoidal formula, when used to obtain the volumes of frustums of pyramids and cones, saves the labor of extracting a square root, as required by the ordinary rule.

EXAMPLE.—The upper and lower bases of the frustum of a right cone are respectively 30 inches and 14 inches in diameter, and the perpendicular distance between them is 36 inches. Required, the volume of the frustum.

SOLUTION.—The diameter of the middle section is the mean of the diameters of the ends, or $\frac{30+14}{2} = 22$ inches.

Area of lower base $= A = .7854 \times 30^2 = 706.86$ sq. in.
Area of upper base $= a = .7854 \times 14^2 = 153.94$ sq. in.
Area of middle section $= M = .7854 \times 22^2 = 380.13$ sq. in.

Using the prismoidal formula,

$$V = \frac{36}{6}(706.86 + 153.94 + 4 \times 380.13) = 14{,}287.92 \text{ cu. in.} = 8.27 \text{ cu. ft.}$$

Ans.

THE SPHERE.

149. A **sphere** is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within called the center. Fig. 109. The word *ball* is commonly used instead of sphere.

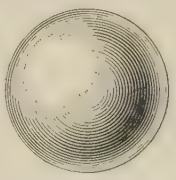

FIG. 109.

150. **Rule.**—*To find the area of the surface of a sphere, multiply the square of the diameter by 3.1416.*

Let d = diameter of sphere;
S = surface.

Then, $S = \pi d^2 = 3.1416\, d^2$.

EXAMPLE.—A ball on a flagstaff is 10 inches in diameter and is to be gilded. Deducting 20 square inches for space covered by attachment to pole, how many books of gold leaf, each containing 25 leaves $3\frac{3}{8}$ inches square, will be required for gilding it?

SOLUTION.—Applying the formula, $S = \pi d^2 = 3.1416 \times 10 \times 10 = 314.16$ square inches. Deducting 20 square inches, the net surface is 294.16 square inches. Each gold leaf has an area of $3\frac{3}{8}$ in. $\times 3\frac{3}{8}$ in., or 11.4 square inches, nearly; hence for 294.16 square inches, there will be needed $294.16 \div 11.4 = 25.8$ leaves, or 1 book and 1 leaf. Ans.

151. **Rule.**—*To find the volume of a sphere, multiply the cube of the diameter by .5236.*

Let d = diameter;
V = volume.

Then, $$V = \frac{\pi}{6} d^3 = .5236\, d^3.$$

Since $\frac{\pi d^3}{6} = \frac{1}{6} \times \pi d^2 \times d$, the volume of a sphere is equal to one-sixth of the surface multiplied by the diameter.

EXAMPLE.—What is the weight of a cast-iron ball 6 inches in diameter, if the metal weighs .26 pound per cubic inch?

SOLUTION.—Applying the formula,

$V = .5236\, d^3 = .5236 \times 216 = 113.1$ cubic inches.

The weight is, therefore, $113.1 \times .26 = 29.41$ lb. Ans.

152. The volume of a spherical shell is equal to the difference in volume between two spheres having the outer and inner diameters of the shell.

153. **Rule.**—*To find the diameter of a sphere of known volume, divide the volume by .5236 and extract the cube root of the quotient.*

$$d = \sqrt[3]{\frac{V}{.5236}}.$$

EXAMPLE.—A cast-iron ball weighing 25 pounds is required for a certain purpose. If cast iron weighs .26 pound per cubic inch, what will be the diameter of the ball?

SOLUTION.—The volume of the ball will be $25 \div .26 = 96.1$ cubic inches. Using the formula,

$$d = \sqrt[3]{\frac{V}{.5236}} = \sqrt[3]{\frac{96.1}{.5236}} = \sqrt[3]{183.5} = 5.68 \text{ inches} = 5\tfrac{11}{16} \text{ in., nearly.}$$

Ans.

SYMMETRICAL AND SIMILAR FIGURES.

154. An **axis of symmetry** is any line so drawn that, if the part of the figure on one side of the line be folded on this line, it will coincide exactly with the other part, point for point and line for line. Thus, in Fig. 110, if the upper semicircle be folded over on the diameter CD, it will coincide exactly with the lower semicircle; also, if the part on the right of the diameter AB be folded over on AB, it will coincide exactly with the part of the left of this line.

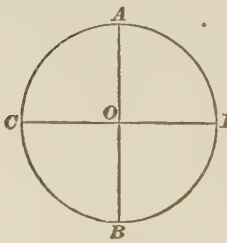

FIG. 110.

It is evident from the above that a circle may have any number of axes of symmetry. In certain cases, however, a figure may be symmetrical with regard to only one axis. Thus, the isosceles triangle $A\,B\,C$, Fig. 111, is symmetrical with regard to the axis $B\,D$, because the part $B\,C\,D$ would coincide with the part $B\,A\,D$ if folded over on the line $B\,D$; but no other axis of symmetry could be drawn. A rectangle has two axes of symmetry at right angles to each other. A hexagon has six axes of symmetry.

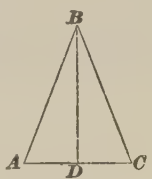

FIG. 111.

155. **Similar figures** are those which are alike in form. As in the case of triangles, which have been considered, two figures, to be similar, must have their corresponding sides in proportion, and the angles of one equal to the corresponding angles of the other. Circles are, of course, similar figures.

156. *The areas of two similar figures are to each other as the squares of any one dimension.* Thus, a regular octagon whose sides are 1 inch long contains 4.828 square inches; another with sides 4 inches long contains 4^2 or 16 times 4.828 square inches = 77.25 square inches, for let A = required area; then A : 4.828 sq. in. $= 4^2 : 1^2$, or $A = 4.828 \times 16$ square inches.

This principle often saves considerable labor in determining the areas of similar figures, as, for instance, the end areas of frustums of pyramids, cones, etc.

EXAMPLE.—Required, the volume of the frustum of a pyramid, the bases of which are equilateral triangles, one with sides 6 feet in length, the other with sides 2 feet in length. The altitude of the frustum is 8 feet.

SOLUTION.—The area of the lower triangular base may be found from the formula $A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, where $s = \frac{6+6+6}{2} = 9$. and the factors $(s-a)$, etc. are each $9-6=3$.

Hence, $A = \sqrt{9\times3\times3\times3} = 15.59$ square feet.

Since the ends are similar triangles, their areas are proportional to the squares of the sides; or, denoting the area of the upper base by a, $a : 15.59 = 2^2 : 6^2$.

Hence, $a = 15.59 \times \frac{2^2}{6^2} = 15.59 \times \frac{1}{9} = 1.73$ square feet.

A section midway between the bases is evidently, also, an equilateral triangle with sides $\frac{6+2}{2} = 4$ feet in length; hence, its area may be obtained from the proportion

$$M : 15.59 = 4^2 : 6^2,$$

or $$M = 15.59 \times \frac{4^2}{6^2} = 6.93 \text{ square feet.}$$

Now using the prismoidal formula,

$$V = \tfrac{8}{6}(15.59 + 1.73 + 4 \times 6.93) = 60.05 \text{ cu. ft.} \quad \text{Ans.}$$

157. The cubical contents (and weights) of similar solids are to each other as the cubes of any one dimension.

EXAMPLE.—If a cast-iron ball 9 inches in diameter weighs 100 pounds, what would a ball 15 inches in diameter weigh?

SOLUTION.— $100 : x = 9^3 : 15^3$, or $x = \frac{100 \times 3{,}375}{729} = 462.96$ pounds, the weight of the larger ball. Ans.

EXAMPLES FOR PRACTICE.

(*See note at bottom of page 12.*)

158. Solve the following examples:

1. What is the weight of the cast-iron lintel shown in Fig. 112, estimating iron at .26 pound per cubic inch? Ans. 201.08 lb.

SUGGESTION.—The lintel consists of a rectangular base, a longitudinal rib whose faces are trapezoids, and two triangular cross-ribs.

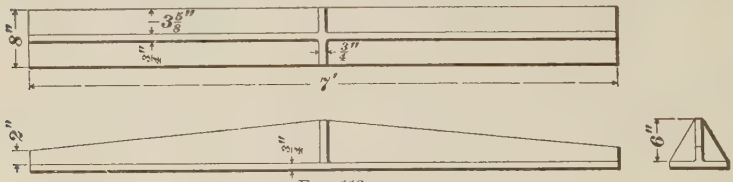

FIG. 112.

2. (*a*) Figure the number of feet B. M. (board measure) in the following bill of material, 1 foot B. M. being 1 foot square and 1 inch thick; (*b*) also the cost at $16.50 per M. (thousand) feet.

23 pieces 3″ × 10″; 18′ 6″ long.	2 pieces 3″ × 6″; 10′ 6″ long.
3 pieces 3″ × 10″; 15′ 0″ long.	5 pieces 3″ × 8″; 18′ 6″ long.
3 pieces 3″ × 10″; 12′ 0″ long.	56 pieces 2″ × 4″; 9′ 9″ long.
34 pieces 3″ × 6″; 14′ 0″ long.	81 pieces 2″ × 4″; 9′ 3″ long.
4 pieces 3″ × 6″; 6′ 0″ long.	42 pieces 2″ × 4″; 1′ 2″ long.
38 pieces 3″ × 4″; 5′ 6″ long.	

SUGGESTION.—An easy method to figure B. M. is as follows: Note that in the first item, a board 10 inches wide and 1 foot long contains

$\frac{10}{12}$ or $\frac{5}{6}$ of a board foot for each inch of thickness. Hence, a $3'' \times 10''$ plank contains $\frac{5}{6} \times 3 = 2.5$ feet, B. M. per foot of length.

Ans. { (*a*) 3,338 ft. B. M., nearly.
(*b*) $55.08.

3. What is the weight of the boiler front shown in Fig. 113 if the metal is $1\frac{1}{4}$ inches thick, and cast iron weighs .26 pound per cubic inch? Ans. 1,638 lb.

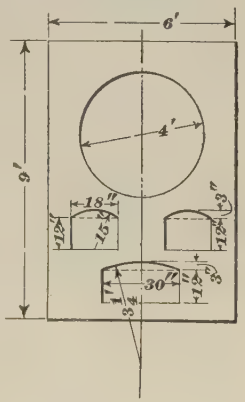

FIG. 113.

SUGGESTION.—First find net area of surface; door openings consist of rectangular portion + circular segment whose chord and height are known.

4. Figure the cost of digging and lining a well 24 feet deep and $3\frac{1}{2}$ feet clear diameter, the lining to be 1 brick thick, laid without mortar; allow 18 brick per square foot of surface of interior of lining. The excavation is to be $5\frac{1}{4}$ feet in diameter. The digging is to cost $1.25 per cubic yard, and the brickwork $12 per thousand, furnished and laid. Ans. $81.04.

5. Find the diameter of the balls of a 10-pound dumbbell, the cylindrical handle of which is $4\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches in diameter. Cast iron weighs .26 pound per cubic inch. Disregard the small volumes at ends of handle, which are figured twice. Ans. 3.16 in. $= 3\frac{1}{8}+$ in.

SUGGESTION.—Find contents of handle; one-half of remainder is volume of each ball.

6. A window weighing 10 pounds is to be balanced by two $1\frac{1}{2}$-inch (inside diameter) lead-pipe weights. If the metal is $\frac{1}{4}$ inch thick, and lead weighs .41 pound per cubic inch, how long pieces are required? Ans. 8.9 in., say 9 in.

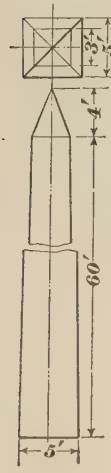

FIG. 114.

SUGGESTION.—Find number of cubic inches in each weight. Length required = volume ÷ area of ring.

7. A shop 28 ft. $\times$ 40 ft. and 18 ft. high, with gables 8 ft. 8 in. high, is to be built of brick. The lower walls are 12 inches, and the gables 8 inches in thickness. Deduct 12 windows 3 ft. $\times$ 6 ft. 4 in., and 4 doors 4 ft. $\times$ 7 ft., all in 12-inch wall. Estimating 7 brick to each square foot of wall *4 inches thick*, and counting corners twice, how many thousand brick will be required, with 5 per cent. allowance for breakage, etc.? Ans. 50.05 thousand.

SUGGESTION.—Find areas and not volumes.

8. (*a*) What will be the weight of the square granite shaft shown in Fig. 114, estimating the stone at 170 pounds per cubic foot? (*b*) Allowing 4,000 pounds per square foot on the soil, what will be the size of the foundation required?

Ans. { (*a*) 168,640 lb. = 84.3 T.
(*b*) 6.5 ft. square, nearly.

Query.—What two solids make up the monument?

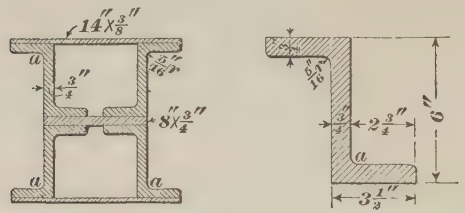

FIG. 115.

9. What is the weight per foot of the **Z** bar column shown in Fig. 115, figuring steel at .283 pound per cubic inch? Neglect rounding of edges of **Z** bars, but figure the fillets, as at *a*. Check result by multiplying area of section by 3.4, which is the weight per foot of steel for each square inch of section.

Ans. { 173.67 lb.
{ 173.88 lb.

SUGGESTION.—Area of a fillet = $\frac{1}{4}$ difference between square of twice the radius, and circle.

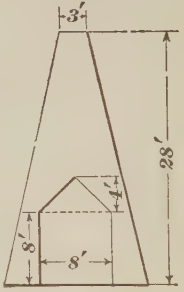

FIG. 116.

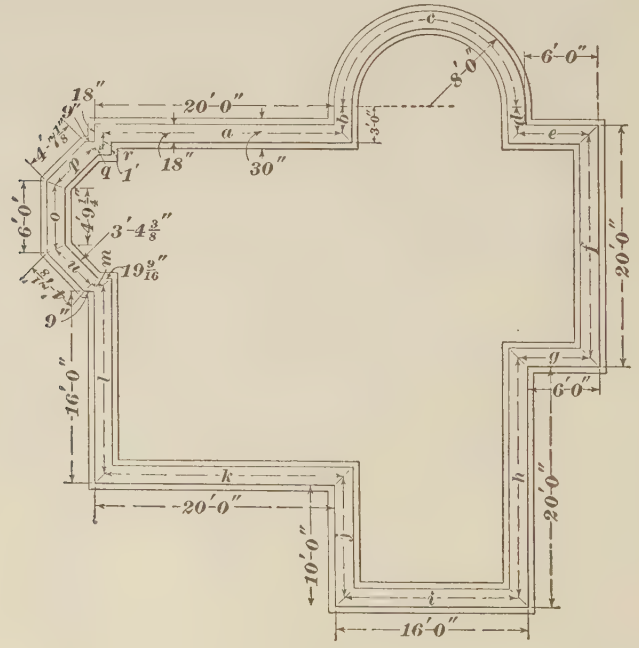

FIG. 117.

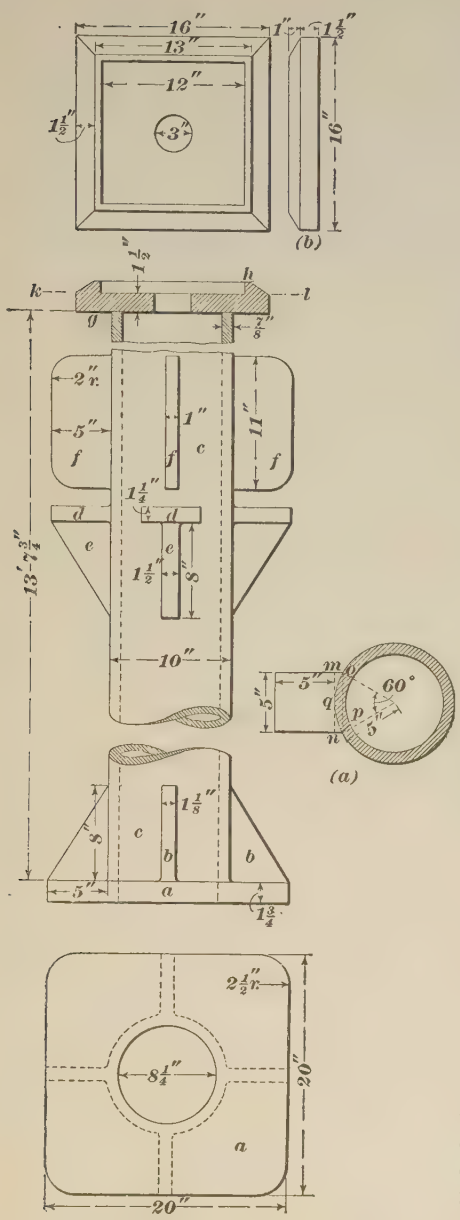

FIG. 118.

10. What is the net area of the 4-sided pyramidal-tower roof shown in Fig. 116, the openings being measured along the slope?

Ans. 710.68 sq. ft.

11. Find the number of cubic yards of masonry in the foundation walls shown in Fig. 117, the wall being 18 inches wide and 9 feet high, and the footing course 2 ft. 6 in. wide and 6 inches thick, projecting 6 inches each side of the foundation wall.

Ans. 95.73 cu. yd., nearly.

SUGGESTION.— $19\frac{9}{16}$ in. = 1.63 ft.; reduce feet and inches to feet and decimals by conversion table. Find the length of center line around the walls; for example, $b = 18$ in. $+ \frac{1}{2}$ the thickness of wall; $f = 20$ ft. $-$ thickness of wall; o = mean of inside and outside measurements, etc. The same center line applies to both footing and wall.

12. Figure the volume of each portion and also the total weight of the cast-iron column shown in Fig. 118, the metal to be taken at .26 pound per cubic inch. Deduct for rounded corners, but figure d as 5 inches square on top.

ANS.—Base, a, 597.06 cu. in.; 4 triangular ribs, b, 90 cu. in.; cylinder, c, 4,106.85 cu. in.; 4 brackets, $d + e$, 245 cu. in.; 4 lugs, f, 213.16 cu. in.; portion of cap below

k l, 373.39 cu. in.; portion above *k l*, 67 cu. in.; total, 5,692.46 cu. in. Weight, 1,480 lb.

SUGGESTION.—Deduction of each rounded corner $= \frac{1}{4}$ of difference between a square of twice the radius and a circle of the given radius. Top of caps above line *k l* = (16 in. square − 12 in. square) × 1 in. thick − 4 wedges, 16 in. × 1 in. at back and 13 in. at edge and $1\frac{1}{2}$ in. long. See sketch (*b*). Area of top of *d* is strictly more than 5 in. × 5 in. by areas *m o q* and *q n p* [see sketch (*a*)] = difference between rectangle *m o p n* and segment of circle *o q p*. Although not figured in the example, the student should calculate it, as a test of his knowledge.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions and examples contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, and that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

ARITHMETIC.

(ARTS. 1–181. SEC. 1.)

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words:
(*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(6) Represent in figures the following expressions:
(*a*) Seven thousand six hundred. (*b*) Eighty-one thousand four hundred two. (*c*) Five million four thousand seven. (*d*) One hundred eight million ten thousand one. (*e*) Eighteen million six. (*f*) Thirty thousand ten.

(7) What is the sum of $3,290 + 504 + 865,403 + 2,074 + 81 + 7$?
Ans. 871,359.

(8) $709 + 8,304,725 + 391 + 100,302 + 300 + 909 =$ what?
Ans. 8,407,336.

(9) Find the difference between the following:
(*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.
Ans. { (*a*) 47,624. (*b*) 5,338.

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(10) (*a*) $70,968 - 32,975 = ?$ (*b*) $100,000 - 98,735 = ?$

Ans. { (*a*) 37,993. (*b*) 1,265.

(11) The greater of two numbers is 1,004 and their difference is 49; what is their sum? Ans. 1,959.

(12) From $5,962 + 8,471 + 9,023$ take $3,874 + 2,039$.

Ans. 17,543.

(13) A man willed $125,000 to his wife and two children; to his son he gave $44,675, to his daughter $26,380, and to his wife the remainder. What was his wife's share?

Ans. $53,945.

(14) Find the products of the following:

(*a*) $526,387 \times 7$; (*b*) $700,298 \times 17$; (*c*) $217 \times 103 \times 67$.

Ans. { (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(15) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800 times.

(16) If a monthly publication contains 24 pages in each issue, how many pages will there be in 8 yearly volumes?

Ans. 2,304.

(17) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) $34,689. (*b*) $37,935.

(18) Solve the following by cancelation:

(*a*) $\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$ (*b*) $\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$

Ans. { (*a*) 8. (*b*) 32.

(19) If a mechanic earns \$1,500 a year for his labor, and his expenses are \$968 per year, in what time can he save enough to buy 28 acres of land at \$133 an acre?

Ans. 7 yr.

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 mi.

(21) If the driving wheel of a locomotive is 16 feet in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance between which is 354 miles, there being 5,280 feet in one mile?

Ans. 116,820 rev.

(22) What is the quotient of:

(*a*) 589,824 ÷ 576? (*b*) 369,730,620 ÷ 43,911? (*c*) 2,527,525 ÷ 505? (*d*) 4,961,794,302 ÷ 1,234?

Ans. (*a*) 1,024. (*b*) 8,420. (*c*) 5,005. (*d*) 4,020,903.

(23) A man paid \$444 for a horse, wagon, and harness. If the horse cost \$264 and the wagon \$153, how much did the harness cost? Ans. \$27.

(24) What is the product of:

(*a*) 1,024 × 576? (*b*) 5,005 × 505? (*c*) 43,911 × 8,420?

Ans. (*a*) 589,824. (*b*) 2,527,525. (*c*) 369,730,620.

(25) If a man receives 30 cents an hour for his wages, how much will he earn in a year, working 10 hours a day and averaging 25 days per month? Ans. \$900.

(26) What is a fraction?

(27) What are the terms of a fraction?

(28) What does the denominator show?

(29) What does the numerator show?

(30) How do you find the value of a fraction?

(31) Is $1\frac{3}{8}$ a proper or an improper fraction, and why?

(32) Write three mixed numbers.

(33) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(34) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(35) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions.
Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(36) What is the value of each of the following: $\frac{13}{2}$, $\frac{17}{4}$, $\frac{69}{16}$, $\frac{16}{8}$, $\frac{67}{64}$? Ans. $6\frac{1}{2}$, $4\frac{1}{4}$, $4\frac{5}{16}$, 2, $1\frac{3}{64}$.

(37) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. (*a*) 112. (*b*) $\frac{3}{16}$. (*c*) $\frac{17}{18}$. (*d*) $4\frac{1}{28}$. (*e*) $3\frac{3}{5}$.

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = ?$ Ans. 1.

(39) $\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ Ans. $\frac{15}{16}$.

(40) $42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Ans. $83\frac{1}{16}$.

(41) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches; and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?
Ans. $190\frac{9}{16}$ sq. in.

(42) Find the value of each of the following:

(*a*) $\dfrac{\frac{7}{3}}{16}$; (*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}}$; (*c*) $\dfrac{\frac{4+3}{2+6}}{5}$. Ans. (*a*) $37\frac{1}{3}$. (*b*) $\frac{3}{4}$. (*c*) $\frac{7}{40}$.

(43) The numerator of a fraction is 28, and the value of the fraction $\frac{7}{8}$; what is the denominator? Ans. 32.

(44) What is the difference between (*a*) $\frac{7}{8}$ and $\frac{7}{16}$? (*b*) 13 and $7\frac{7}{16}$? (*c*) $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. { (*a*) $\frac{7}{16}$. (*b*) $5\frac{9}{16}$. (*c*) $83\frac{13}{32}$.

(45) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles in another day, and $125\frac{17}{35}$ miles in another day, how far did he travel in the three days?

Ans. $289\frac{211}{420}$ mi.

(46) From $573\frac{4}{5}$ tons take $216\frac{5}{8}$ tons. Ans. $357\frac{7}{40}$ T.

(47) At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $9\frac{1}{4}$ yards of cloth? Ans. $3\frac{15}{32}$ dollars.

(48) Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 by $\frac{7}{8}$ of $\frac{5}{6}$ of 45.

Ans. $109\frac{13}{128}$.

(49) How many times are $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

Ans. 18 times.

(50) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{1}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left?

Ans. $52\frac{13}{16}$ lb.

(51) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(52) How do you place decimals for addition and subtraction?

(53) Give a rule for multiplication of decimals.

(54) Give a rule for division of decimals.

(55) State the difference between a fraction and a decimal.

(56) State how to reduce a fraction to a decimal.

(57) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{8^2}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. .5. .875. .15625. .65. .125.

(58) Solve the following:

(*a*) $\dfrac{32.5+.29+1.5}{4.7+9}$; (*b*) $\dfrac{1.283\times\overline{8+5}}{2.63}$;

(*c*) $\dfrac{\overline{589+27}\times\overline{163-8}}{25+39}$; (*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}$.

Ans. (*a*) 2.5029. (*b*) 6.3418. (*c*) 1,491.875. (*d*) 8.1139.

(59) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

(60) What decimal part of a foot is $\frac{3}{16}$ of an inch?

Ans. .015625.

(61) A cubic inch of water weighs .03617 of a pound. What is the weight of a body of water whose volume is 1,500 cubic inches? Ans. 54.255 lb.

(62) If by selling a carload of coal for $82.50, at a profit of $1.65 per ton, I make enough to pay for 72.6 feet of fencing at $.50 a foot, how many tons of coal were in the car? Ans. 22 T.

(63) Divide 17,892 by 231, and carry the result to four decimal places. Ans. 77.4545+.

(64) What is the value of the following expression carried to three decimal places:

$$\frac{74.26\times24\times3.1416\times19\times19\times350}{33{,}000\times12\times4}=?$$ Ans. 446.619—.

(65) Express: (*a*) .7928 in 64ths; (*b*) .1416 in 32ds; (*c*) .47915 in 16ths.

Ans. (*a*) $\frac{51}{64}$. (*b*) $\frac{5}{32}$. (*c*) $\frac{8}{16}$.

(66) Work out the following examples:

(*a*) $709.63 - .8514$; (*b*) $81.963 - 1.7$; (*c*) $18 - .18$; (*d*) $1 - .001$; (*e*) $872.1 - (.8721 + .008)$; (*f*) $(5.028 + .0073) - (6.704 - 2.38)$.

Ans. (*a*) 708.7786. (*b*) 80.263. (*c*) 17.82. (*d*) .999. (*e*) 871.2199. (*f*) .7113.

(67) Work out the following:

(*a*) $\frac{7}{8} - .807$; (*b*) $.875 - \frac{8}{9}$; (*c*) $(\frac{5}{32} + .435) - (\frac{21}{100} - .07)$; (*d*) What is the difference between the sum of 33 millionths and 17 thousandths, and the sum of 53 hundredths and 274 thousandths?

Ans. (*a*) .068. (*b*) .5. (*c*) .45125. (*d*) .786967.

(68) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(69) $927.416 + 8.274 + 372.6 + 62.07938 = ?$

Ans. 1,370.36938.

(70) Add 17 thousandths, 2 tenths, and 47 millionths.

Ans. .217047.

(71) Find the products of the following expressions:

(*a*) $.013 \times .107$; (*b*) $203 \times 2.03 \times .203$; (*c*) $2.7 \times 31.85 \times (3.16 - .316)$; (*d*) $(107.8 + 6.541 - 31.96) \times 1.742$.

Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702.

(72) Solve the following:

(*a*) $(\frac{7}{16} - .13) \times \overline{.625 + \frac{5}{8}}$; (*b*) $(\frac{19}{32} \times .21) - (.02 \times \frac{3}{16})$; (*c*) $(\frac{13}{4} + .013 - 2.17) \times \overline{13\frac{1}{4} - 7\frac{5}{16}}$.

Ans. (*a*) .384375. (*b*) .1209375. (*c*) 6.4896875.

(73) Solve the following:

(*a*) $.875 \div \frac{1}{2}$; (*b*) $\frac{7}{8} \div .5$; (*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125}$.

Ans. (*a*) 1.75. (*b*) 1.75. (*c*) .5.

(74) Find the value of the following expression:

$$\frac{\dfrac{1.25 \times 20 \times 3}{87 + (11 \times 8)}}{459 + 32}$$

Ans. $210\frac{3}{7}$.

(75) From 1 plus .001 take .01 plus .000001.

Ans. .990999.

ARITHMETIC.

(ARTS. 1–168. SEC. 2.)

(1) What is 25 per cent. of 8,428 lb.? Ans. 2,107 lb.

(2) What is 1 per cent. of \$100? Ans. \$1.

(3) What is $\frac{1}{2}$ per cent. of \$35,000? Ans. \$175.

(4) What per cent. of 50 is 2? Ans. 4%.

(5) What per cent. of 10 is 10? Ans. 100%.

(6) Solve the following:

(*a*) Base = \$2,522 and percentage = \$176.54. What is the rate? (*b*) Percentage = 16.96 and rate = 8 per cent. What is the base? (*c*) Amount = 216.7025 and base = 213.5. What is the rate? (*d*) Difference = 201.825 and base = 207. What is the rate?

Ans. (*a*) 7%. (*b*) 212. (*c*) $1\frac{1}{2}$%. (*d*) $2\frac{1}{2}$%.

(7) A farmer gained 15% on his farm by selling it for \$5,500. What did it cost him? Ans. \$4,782.61.

(8) A man receives a salary of \$950. He pays 24% of it for board, $12\frac{1}{2}$% of it for clothing, and 17% of it for other expenses. How much does he save in a year? Ans. \$441.75.

(9) If $37\frac{1}{2}$ per cent. of a number is 961.38, what is the number? Ans. 2,563.68.

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(10) A man owns $\frac{3}{4}$ of a property. 30% of his share is worth \$1,125. What is the whole property worth?

Ans. \$5,000.

(11) What sum diminished by 35% of itself equals \$4,810?

Ans. \$7,400.

(12) A merchant's sales amounted to \$197.55 on Monday, and this sum was $12\frac{1}{2}$% of his sales for the week. How much were his sales for the week? Ans. \$1,580.40.

(13) The distance between two stations on a certain railroad is 16.5 miles, which is $12\frac{1}{2}$% of the entire length of the road. What is the length of the road? Ans. 132 mi.

(14) After paying 60% of my debts I find that I still owe \$35. What was my whole indebtedness? Ans. \$87.50.

(15) Reduce 28 rd. 4 yd. 2 ft. 10 in. to inches.

Ans. 5,722 in.

(16) Reduce 5,722 in. to higher denominations.

Ans. 28 rd. 4 yd. 2 ft. 10 in.

(17) How many seconds in 5 weeks and 3.5 days?

Ans. 3,326,400 sec.

(18) How many pounds, ounces, pennyweights, and grains are contained in 13,750 gr.?

Ans. 2 lb. 4 oz. 12 pwt. 22 gr.

(19) Reduce 4,763,254 links to miles.

Ans. 595 mi. 32 ch. 54 li.

(20) Reduce 764,325 cu.in. to cu.yd.

Ans. 16 cu.yd. 10 cu.ft. 549 cu.in.

(21) What is the sum of 2 rd. 2 yd. 2 ft. 3 in.; 4 yd. 1 ft. 9 in.; 2 ft. 7 in.? Ans. 3 rd. 2 yd. 2 ft. 1 in.

(22) What is the sum of 3 gal. 3 qt. 1 pt. 3 gi.; 6 gal. 1 pt. 2 gi.; 4 gal. 1 gi.; 8 qt. 5 pt.? Ans. 16 gal. 3 qt. 2 gi.

(23) What is the sum of 240 gr. 125 pwt. 50 oz. and 3 lb.?

Ans. 7 lb. 8 oz. 15 pwt.

(24) What is the sum of 11° 16′ 12″; 13° 19′ 30″; 20° 25″; 26′ 29″; 16° 17′ 11″? Ans. 55° 19′ 47″.

(25) What is the sum of 130 rd. 5 yd. 1 ft. 6 in.; 215 rd. 2 ft. 8 in.; 304 rd. 4 yd. 11 in.? Ans. 2 mi. 10 rd. 5 yd. 7 in.

(26) What is the sum of 21 A. 67 sq.ch. 3 sq.rd. 21 sq.li.; 28 A. 78 sq.ch. 2 sq.rd. 23 sq.li.; 47 A. 6 sq.ch. 2 sq.rd. 18 sq.li.; 56 A. 59 sq.ch. 2 sq.rd. 16 sq.li.; 25 A. 38 sq.ch. 3 sq.rd. 23 sq.li.; 46 A. 75 sq.ch. 2 sq.rd. 21 sq.li.?
Ans. 255 A. 3 sq.ch. 14 sq.rd. 122 sq.li.

(27) From 20 rd. 2 yd. 2 ft. 9 in. take 300 ft.
Ans. 2 rd. 1 yd. 2 ft. 9 in.

(28) From a farm containing 114 A. 80 sq.rd. 25 sq.yd., 75 A. 70 sq.rd. 30 sq.yd. are sold. How much remains?
Ans. 39 A. 9 sq.rd. $25\frac{1}{4}$ sq.yd

(29) From a hogshead of molasses, 10 gal. 2 qt. 1 pt. are sold at one time, and 26 gal. 3 qt. at another time. How much remains? Ans. 25 gal. 2 qt. 1 pt.

(30) If a person were born June 19, 1850, how old would he be August 3, 1892? Ans. 42 yr. 1 mo. 14 da.

(31) A note was given August 5, 1890, and was paid June 3, 1892. What length of time did it run?
Ans. 1 yr. 9 mo. 28 da.

(32) What length of time elapsed from 16 min. past 10 o'clock A. M., July 4, 1883, to 22 min. before 8 o'clock P. M., Dec. 12, 1888? Ans. 5 yr. 5 mo. 8 da. 9 hr. 22 min.

(33) If 1 iron rail is 17 ft. 3 in. long, how long would 51 rails be, if placed end to end? Ans. 53 rd. $1\frac{1}{2}$ yd. 9 in.

(34) Multiply 3 qt. 1 pt. 3 gi. by 4.7.
Ans. 4 gal. 2 qt. 1.7 gi.

(35) Multiply 3 lb. 10 oz. 13 pwt. 12 gr. by 1.5.
Ans. 5 lb. 10 oz. 6 gr.

(36) How many bushels of apples are contained in 9 bbl., if each barrel contains 2 bu. 3 pk. 6 qt.?

Ans. 26 bu. 1 pk. 6 qt.

(37) Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.

Ans. 13 T. 3 cwt. 67.85 lb.

(38) Divide 358 A. 57 sq.rd. 6 sq.yd. 2 sq.ft. by 7.

Ans. 51 A. 31 sq.rd. 8 sq.ft.

(39) Divide 282 bu. 3 pk. 1 qt. 1 pt. by 12.

Ans. 23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt.

(40) How many iron rails, each 30 ft. long, are required to lay a railroad track 23 mi. long? Ans. 8,096 rails.

(41) How many boxes, each holding 1 bu. 1 pk. 7 qt., can be filled from 356 bu. 3 pk. 5 qt. of cranberries?

Ans. 243 boxes.

(42) If 16 square miles are equally divided into 62 farms, how much land will each contain?

Ans. 165 A. 25 sq.rd. 24 sq.yd. 3 sq.ft. 80+ sq.in.

(43) What is the square of 108? Ans. 11,664.

(44) What is the cube of 181.25? Ans. 5,954,345.703125.

(45) What is the fourth power of 27.61?

Ans. 581,119.73780641.

(46) Solve the following: (*a*) 106^2; (*b*) $(182\frac{1}{8})^2$; (*c*) $.005^2$; (*d*) $.0063^2$; (*e*) 10.06^2.

Ans. (*a*) 11,236.
(*b*) 33,169.515625.
(*c*) .000025.
(*d*) .00003969.
(*e*) 101.2036.

(47) Solve the following: (*a*) 753^3; (*b*) 987.4^3; (*c*) $.005^3$; (*d*) $.4044^3$.

Ans. (*a*) 426,957,777.
(*b*) 962,674,279.624.
(*c*) .000000125.
(*d*) .066135317184.

(48) What is the fifth power of 2? Ans. 32.

(49) What is the fourth power of 3? Ans. 81.

(50) What are the values of: (*a*) 67.85^2? (*b*) $967{,}845^2$? (*c*) $(\frac{3}{8})^2$? (*d*) $(\frac{1}{4})^2$?

Ans. (*a*) 4,603.6225. (*b*) 936,723,944,025. (*c*) $\frac{9}{64}$. (*d*) $\frac{1}{16}$.

(51) What is (*a*) the tenth power of 5? (*b*) the fifth power of 9?

Ans. (*a*) 9,765,625. (*b*) 59,049.

(52) Solve the following: (*a*) 1.2^4; (*b*) 11^6; (*c*) 1^7; (*d*) $.01^4$; (*e*) $.1^8$.

Ans. (*a*) 2.0736. (*b*) 1,771,561. (*c*) 1. (*d*) .00000001. (*e*) .00001.

(53) Find the values of the following: (*a*) $.0133^3$; (*b*) 301.011^3; (*c*) $(\frac{1}{8})^3$; (*d*) $(3\frac{3}{4})^3$.

Ans. (*a*) .000002352637. (*b*) 27,273,890.942264331. (*c*) $\frac{1}{512}$. (*d*) $52\frac{47}{64}$, or 52.734375.

(54) In what respect does evolution differ from involution?

NOTE.—In the answers to the following examples, a minus sign after a number indicates that the last digit is not quite as large as the number printed. Thus, 12.497— indicates that the number really is 12.496+, and that the 6 has been made a 7 because the next succeeding figure was 5 or greater. For example, had it been desired to use but three decimal places in example 46 (*b*), the answer would have been written 33,169.516—.

(55) Find the square root of the following: (*a*) 3,486,784.401; (*b*) 9,000,099.4009; (*c*) .001225.

Ans. (*a*) 1,867.29+. (*b*) 3,000.017—. (*c*) .035.

(56) Extract the square root of (*a*) 10,795.21; (*b*) 73,008.04; (*c*) 90; (*d*) .09.

Ans. (*a*) 103.9. (*b*) 270.2. (*c*) 9.487—. (*d*) .3.

(57) Extract the cube root of (*a*) .32768; (*b*) 74,088; (*c*) 92,416; (*d*) .373248.

Ans. (*a*) .6894+. (*b*) 42. (*c*) 45.212—. (*d*) .72.

(58) Extract the cube root of 2 to six decimal places.

Ans. 1.259921+.

(59) Extract the cube root of (*a*) 1,758.416743; (*b*) 1,191,016; (*c*) $\frac{4}{32}$; (*d*) $\frac{27}{512}$.

Ans. (*a*) 12.07. (*b*) 106. (*c*) $\frac{1}{2}$. (*d*) $\frac{3}{8}$.

(60) Extract the cube root of 3 to six decimal places.

Ans. 1.442250—.

(61) Solve the following: (*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$; (*c*) $\sqrt{502,681}$; (*d*) $\sqrt{.00041209}$.

Ans. (*a*) 11.1. (*b*) 10.72+. (*c*) 709. (*d*) .0203.

(62) Solve the following: (*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.021}$; (*c*) $\sqrt[3]{8,036,054,027}$; (*d*) $\sqrt[3]{.000004096}$; (*e*) $\sqrt[3]{17}$.

Ans. (*a*) .18663—. (*b*) .2759—. (*c*) 2,003. (*d*) .016. (*e*) 2.5713—.

(63) Solve the following: (*a*) $\sqrt[4]{6,561}$; (*b*) $\sqrt[6]{117,649}$; (*c*) $\sqrt[6]{.000064}$; (*d*) $\sqrt[3]{\frac{3}{8}}$.

Ans. { (*a*) 9. (*b*) 7. (*c*) .2. (*d*) .72112+.

(64) Extract the square root of: (*a*) $\frac{1225}{5476}$; (*b*) .3364; (*c*) .1; (*d*) $25.0\frac{3}{4}$; (*e*) $.000\frac{4}{9}$.

Ans. { (*a*) $\frac{35}{74}$. (*b*) .58. (*c*) .31623−. (*d*) 5.00749+. (*e*) .02108+.

(65) (*a*) Extract the fourth root of 2 to four decimal places. (*b*) Extract the sixth root of 6.

Ans. { (*a*) 1.1892+. (*b*) 1.34801−.

(66) Extract the square root of (*a*) 3.1416 and (*b*) .7854 to four decimal places.

Ans. { (*a*) 1.7725−. (*b*) .8862+.

(67) Extract the cube root of (*a*) 3.1416 and (*b*) .5236 to four decimal places.

Ans. { (*a*) 1.4646−. (*b*) .8060−.

Find the value of x in the following:

(68) $11.7 : 13 :: 20 : x$. Ans. 22.22+.

(69) (*a*) $20+7 : 10+8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. { (*a*) 2. (*b*) 277.7+.

(70) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. { (*a*) $x = 12$. (*b*) $x = 12$. (*c*) $x = 20$. (*d*) $x = 180$. (*e*) $x = 40$.

(71) $x : 5 :: 27 : 12.5$. Ans. $10\frac{4}{5}$.

(72) $45 : 60 :: x : 24$. Ans. 18.

(73) $x : 35 :: 4 : 7$. Ans. 20.

(74) $9 : x :: 6 : 24$. Ans. 36.

(75) $\sqrt[3]{1,000} : \sqrt[3]{1,331} = 27 : x$. Ans. 29.7.

(76) $64 : 81 = 21^2 : x^2$. Ans. 23.625.

(77) $7 + 8 : 7 = 30 : x$. Ans. 14.

(78) A man whose steps measure 2 ft. 5 in. takes 2,480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

Ans. 2,320 steps.

(79) If a horse travels 12 mi. in 1 hr. 36 min., how far will he travel at the same rate in 15 hr.? Ans. 112.5 mi.

(80) If a column of mercury 27.63 in. high weighs .76 of a pound, what will be the weight of a column of mercury having the same diameter, 29.4 in. high? Ans. .808+ lb.

(81) If 2 gal. 3 qt. 1 pt. of water will last a man 5 da., how long will 5 gal. 3 qt. last him, if he drinks at the same rate?

Ans. 10 da.

(82) Heat from a burning body varies inversely as the square of the distance from it. If a thermometer held 6 ft. from a stove shows a rise in temperature of 24°, how many degrees rise in temperature would it indicate if held 12 ft. from the stove? Ans. 6°.

(83) If a pile of wood 12 ft. long, 4 ft. wide, and 3 ft. high is worth $12, what is the value of a pile of wood 15 ft. long, 5 ft. wide, and 6 ft. high? Ans. $37.50.

(84) If 100 gal. of water run over a dam in 2 hr., how many gallons will run over the dam in 14 hr. 28 min.?

Ans. $723\frac{1}{3}$ gal.

(85) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep?

Ans. $484\frac{1}{2}$ bbl.

(86) If a railway train runs 444 mi. in 8 hr. 40 min., in what time can it run 1,060 mi. at the same rate of speed?

Ans. 20 hr. 41.44 min.

(87) If sound travels at the rate of 6,160 ft. in $5\frac{1}{2}$ sec., how far does it travel in 1 min.? Ans. 67,200 ft.

(88) If 5 men by working 8 hours a day can do a certain amount of work, how many men by working 10 hours a day can do the same work? Ans. 4 men.

(89) If a man travels 540 miles in 20 days of 10 hours each, how many hours a day must he travel to cover 630 miles in 25 days? Ans. $9\frac{1}{3}$ hr.

(90) Referring to example 4, Art. **168,** *Arithmetic*, § 2, what is the horsepower of an engine whose cylinder is 30 inches in diameter, piston speed, 660 feet per minute, and mean effective pressure, 42 pounds per square inch?

Ans. 594 horsepower.

(91) The weight of a cubic inch of cast iron is .261 pound. Referring to Art. **164,** *Arithmetic*, § 2, what is the weight of a solid cast-iron cylinder whose diameter is 12 inches and length is 60 inches? Ans. 1,771.11 lb.

(92) Referring to Art. **167,** *Arithmetic*, § 2, what is the centrifugal force of a 40-pound body revolving in a circle having a radius of 10 inches, at a speed of 18 feet per second? Ans. 484.7 lb.

FORMULAS.

(ARTS. 1–21. SEC. 3.)

$A = 5$ $\quad h = 200$
$B = 10$ $\quad x = 12$
$i = 3.5$ $\quad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(1) $C = \dfrac{D - x}{B + i}$. Ans. $C = 8$.

(2) $Q = \dfrac{A\,h + D}{2\,x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(3) $r = \dfrac{3.246\,B\,h}{\dfrac{A\,x + h}{A\,i - B}}$. Ans. $r = 187.269+$.

(4) $v = \sqrt{\dfrac{A\,D}{i\,B + 1.5}}$. Ans. $v = 4.05+$.

(5) $u = \sqrt[3]{\dfrac{B\,x}{.00018\,h(A^2 - x)}}$. Ans. $u = 6.35+$.

(6) $f = \dfrac{10(h - D)^2}{\sqrt[3]{D + A}}$. Ans. $f = 12{,}800$.

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(7) $g = \dfrac{(B-A)^2 - \sqrt[3]{D+A}}{A^3-(1+D)}$. Ans. $g = 5$.

(8) $k = \sqrt{\dfrac{A\,B^2}{\sqrt[3]{A\,h}}}$. Ans. $k = 7.071+$.

(9) $T = \sqrt{\dfrac{A^2\left[490 + \dfrac{(h\,x)^2}{D^2}\right]}{h + \dfrac{x}{D}(A^2-B)^2}}$. Ans. $T = 10$

GEOMETRY AND MENSURATION.

(ARTS. 1-158. SEC. 4.)

(See note at bottom of page 12, Instruction Paper.)

(1) Fig. 1 represents a center—half a regular hexagon—for a 5-foot semicircular arch. (*a*) What is the angle of bevel (*A B O*) to which the ends of the planks must be cut? (*b*) What length of plank, as *F D*, is needed for each piece? (*c*) What is the length of the inner side, as *A B*, of each plank, after cutting, if the width is 8 inches?

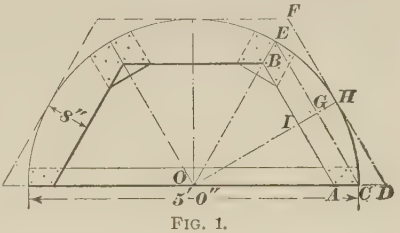

FIG. 1.

Ans. { (*a*) 60°. (*b*) 2.89 ft. = 2 ft. 10.7 in. (*c*) 2.12 ft. = 2 ft. 1½ in.

(2) How many feet of molding are needed to go around an 8-sided room, in which the distance between the parallel sides is 12 feet, and between the opposite angles, 13 feet?

Ans. 40 ft.

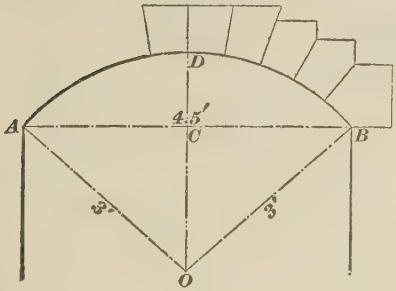

FIG. 2.

(3) *A D B*, Fig. 2, represents an arch of 3 feet radius. (*a*) The span *A B* being 4½ feet, what is the

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rise CD? (b) If the radius OA, and rise CD were given, show how to find AB. Ans. (a) 1.02 ft. = 1 ft. $\frac{1}{4}$ in.

(4) In staking out a building, the points A, B, C, D, E, and F are located, the distances being as shown in Fig. 3. To check these measurements, it is desired to measure the diagonals FB, FD, EA, and EC. What are their lengths? Ans. $FB = 30$ ft.; $FD = 33.11$ ft.; $EA = 38.42$ ft.; $EC = 18.44$ ft.

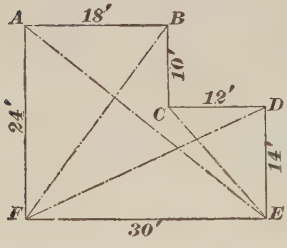

FIG. 3.

(5) Explain why the sides of a regular hexagon are equal to the radius of the circumscribing circle.

(6) In a half-pitch (or 45°) roof of 24-foot span, show that the length along the roof from side wall to ridge = $\sqrt{288}$ feet.

(7) Show by a sketch how to divide a line 6 inches long into 7 equal parts.

(8) In Fig. 4, what is the length of the ridge plate AB, which is 10 feet above CD, the latter being 36 feet long, and the angles DCA and CDB being 45°? Ans. 16 ft.

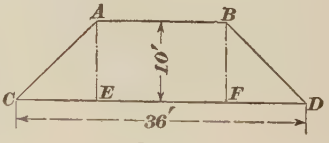

FIG. 4.

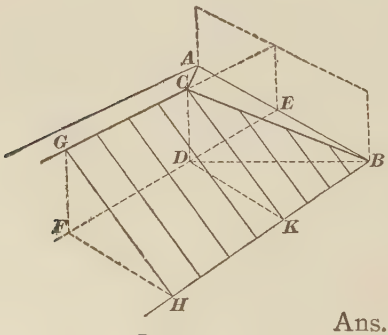

FIG. 5.

(9) Fig. 5 represents a roof whose span AB is 22 feet. The slopes are half-pitch, and the distance from E to D (under C) is 11 feet. (a) Find the length of a "common rafter," as GH. (b) Find the length of a "hip rafter," as BC.

Ans. (a) 15.56 ft. = 15 ft. $6\frac{3}{4}$ in.
(b) 19.05 ft. = 19 ft. $\frac{5}{8}$ in.

(10) In Fig. 6, the distances from A to C, and from B to D, across a stream, are required. The line $A\,B$ is 210 feet long, and E is the middle point. $E\,F$, parallel to $B\,D$, is 86 feet long; and $E\,G$, parallel to $A\,C$, is 90 feet long. What are the lengths of $A\,C$ and $B\,D$?

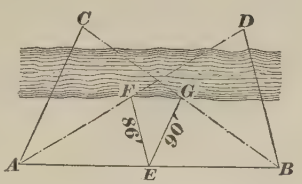

FIG. 6.

Ans. { $A\,C$, 180 ft. / $B\,D$, 172 ft.

(11) In a right triangle, one of the acute angles is 37°. (*a*) What is the other acute angle? (*b*) What is the angle formed by producing one side of the given angle?

Ans. { (*a*) 53°. / (*b*) 143°.

(12) What is the angle included between two adjacent sides of a regular nonagon (nine-sided polygon)?

Ans. 140°.

(13) In Fig. 7, the span $A\,C$ of a semicircular arch is 16 feet, and the distance $A\,E$ to the top of the masonry is 11 feet. How thick is the stonework at $D\,F$, 4 feet from the vertical line $A\,E$? Ans. 4.07 ft.

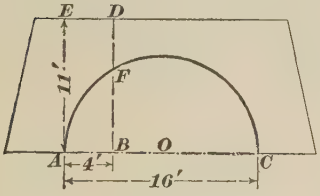

FIG. 7.

(14) Show how to lay out, with a tape, a line 12 feet long, perpendicular to another at its middle point, the length of the latter being 32 feet.

(15) The corners of a cast-iron plate, 15 inches square and $1\frac{1}{4}$ inches thick, are rounded off by quarter circles of 2-inch radius. Estimating cast iron at .26 pound per cubic inch, how much less does this plate weigh than if the corners were square? Ans. 1.11 lb.

(16) A brick pier is 30 inches square at the base, 18 inches square at the top, and 6 feet high. Figuring $22\frac{1}{2}$ brick per cubic foot, how many brick are required for 18 piers? Ans. 9,922.5.

(17) Fig. 8 represents a cross-section of a "Phœnix" column, made of wrought iron, of which a strip 1 foot long and 1 inch square in section weighs 3.33 pounds. Disregarding curved corners and edges, but adding 2 per cent. to the weight of the iron for rivet heads, what is the weight of the column per foot of length?

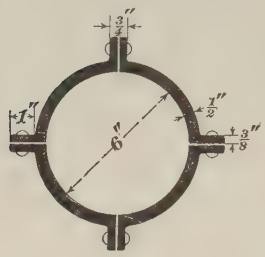

FIG. 8.

Ans. 44.87 lb.

(18) The freight rate from a sandstone quarry to a town is 21 cents per hundred pounds. The following pieces of dimension stone were shipped. Allowing 140 pounds to the cubic foot, what were the charges?

1 piece 4′ 6″ × 3′ 9″ × 15″. 2 pieces 5′ 6″ × 4′ 0″ × 15″.
1 piece 4′ 3″ × 3′ 6″ × 15″. 1 piece 5′ 0″ × 3′ 9″ × 15″.
1 piece 5′ 3″ × 4′ 0″ × 15″. 1 piece 4′ 9″ × 4′ 6″ × 15″.
3 pieces 3′ 3″ × 3′ 6″ × 15″. 2 pieces 4′ 6″ × 4′ 3″ × 15″.

Ans. $76.90.

(19) Fig. 9 represents the plan and end views of a roof.

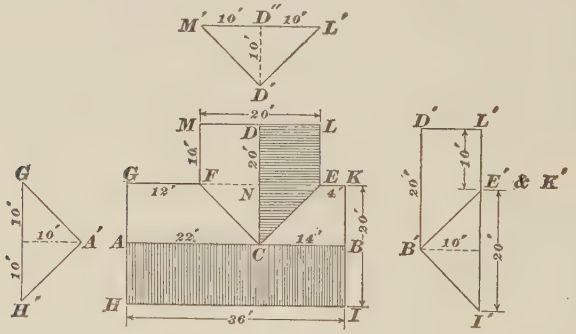

FIG. 9.

Knowing that the ridges $A\,B$ and $C\,D$ are 10 feet above the eaves, find (*a*) the total area of the roof, and (*b*) the length of the "valleys" $F\,C$ and $E\,C$.

Ans. { (*a*) 1,300.88 sq. ft.
(*b*) 17.32 ft. = 17 ft. $3\frac{7}{8}$ in.

(20) Figuring $7\frac{1}{2}$ gallons per cubic foot, how many gallons will be discharged per minute through a 4-inch pipe, if the water flows 5 feet per second? Take the area of pipe as .09 square foot. Ans. 202.5 gal.

(21) What length of lead pipe, weighing .41 pound per cubic inch, $1\frac{3}{4}$ inches outside diameter and $\frac{1}{8}$ inch thick, will be needed to make a 3-pound weight? **Ans. 11.6 in.**

(22) The span $A\,B$ of the arch shown in Fig. 10 is 6 feet 8 inches, and the rise $C\,D$ is 8 inches. (*a*) Find the radius. (*b*) If there are 13 ring stones, what is the bottom width of each?

Ans. { (*a*) 8 ft. 8 in. (*b*) 6.32 in.

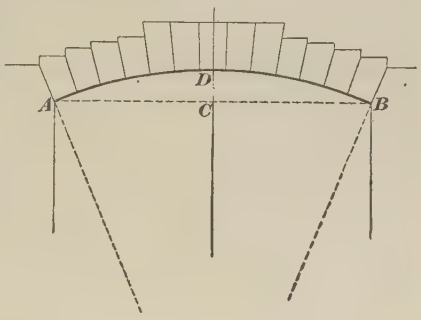

FIG. 10.

(23) In Fig. 10, find the radius by the principle that a perpendicular from any point on a circumference to a diameter is a mean proportional between the two parts of the diameter.

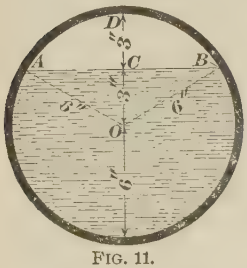

FIG. 11.

(24) It is required, in a question of water supply, to determine the area of the water section in a 12-inch pipe flowing 9 inches deep; that is, what is the area below the surface $A\,B$, Fig. 11? (See Art. **106.**) Ans. 91.11 sq. in.

(25) Find the total feet B. M. in the following bill of material:

3 pieces 3″ × 10″; 16′ 0″ long.
8 pieces 3″ × 10″; 12′ 6″ long.
6 pieces 3″ × 8″; 14′ 6″ long.
10 pieces 3″ × 8″; 9′ 0″ long.
4 pieces 3″ × 6″; 10′ 6″ long.
20 pieces 3″ × 6″; 7′ 0″ long.
8 pieces 4″ × 6″; 18′ 0″ long.
4 pieces 4″ × 6″; 6′ 6″ long.
52 pieces 2″ × 4″; 18′ 0″ long.
15 pieces 2″ × 4″; 9′ 6″ long.
600 ft. B. M. 1″ × 6″ flooring.
200 ft. B. M. 2″ plank.

Ans. 2,856 ft. B. M.

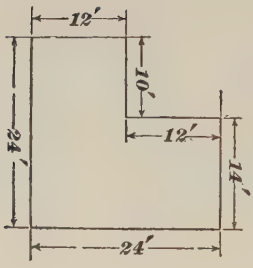

FIG. 12.

(26) Fig. 12 represents the plan of a house which is to be faced with pressed brick. The walls are 22 ft. high, and there are two gables, one 14 ft. × 7 ft. high, and the other 12 ft. × 7 ft. high. Deducting 10 windows, 3 ft. × 7 ft.; 10 windows, 3 ft. × 6 ft.; and 4 doors, 3 ft. 6 in. × 8 ft.; and allowing 7 brick to each square foot of surface, how many brick will be required? Ans. 11,907.

(27) The major and minor axes of an elliptic window frame are 8 feet and 5 feet, respectively. The sash is 4 inches wide all around. What is the area of the glass?

Ans. 24.93 sq. ft.

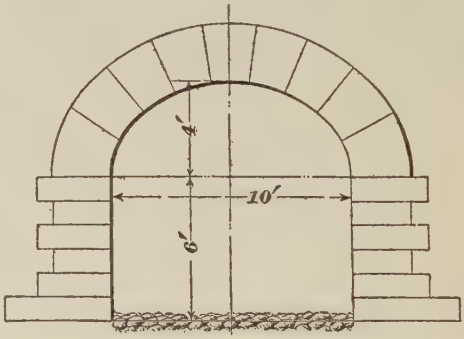

FIG. 13.

(28) A culvert is required to have an area of 88 square feet. How much more than this area is the cross-section shown in Fig. 13, the arch being semi-elliptical? Ans. 3.42 sq. ft.

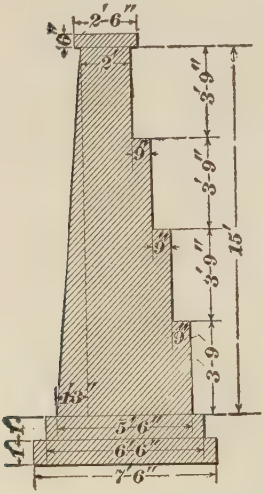

FIG. 14.

(29) How many cubic yards are there in a retaining wall 16 feet long, having the cross-section shown in Fig. 14? Ans. 42.36 cu. yd.

(30) The top of the frame of a window $3\frac{1}{2}$ feet in width is to be bent to a circular arc, the rise of which is 3 inches. What length of piece will be required?

Ans. 42.57 in. = 3 ft. $6\frac{9}{16}$ in.

(31) Compute the cubic yards of excavation for the cellar, 9 feet deep, of the building shown in Fig. 3, increasing all the dimensions, *except* BC and CD, by 2 feet, to give room to lay the masonry. (The reason for not increasing BC and CD will be seen by drawing lines around the plan 1 foot outside the wall lines.) Ans. 237.33 cu. yd.

(32) The length of a building with a plain roof is 32 feet, and the width is 24 feet. The height of the gables is $\frac{5}{8}$ the width. The roof projects over the walls 15 inches (measured on the roof) at ends and eaves. (*a*) How many squares (100 sq. ft.) of slating in the roof area? (*b*) The slates being 12 inches wide and exposed $8\frac{1}{2}$ inches to the weather, how many will be required per square?

Ans. { (*a*) 14.12 squares. (*b*) 141+ slates.

(33) The dome of a cupola is hemispherical, and its diameter is 3 feet. Deducting 5 square feet for windows, etc., what is the area of the remaining surface?

Ans. 9.14 sq. ft.

(34) Estimating cast iron at 450 pounds per cubic foot, what is the weight per 12-foot length of pipe, 10 inches inside diameter, the thickness being $\frac{1}{2}$ inch? Ans. 616.5 lb.

(35) The outside of a circular tower 28 feet high and 8 feet in diameter is to be shingled. (*a*) How many squares (100 sq. ft.) of shingling are required? (*b*) If the shingles average 6 inches wide and are laid 5 inches to the weather, how many will be required to the square?

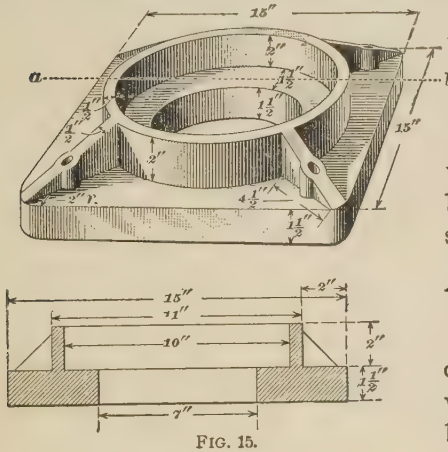

FIG. 15.

Ans. { (*a*) 7.04 squares. (*b*) 480.

(36) At .26 pound per cubic inch, what is the weight of the cast-iron base shown in Fig. 15?

Figure the 4 ribs as plain, with no allowance for swelled parts nor deduction for holes. The corners of the base are rounded to a 2-inch radius. Ans. 82.32 lb.

(37) A bridge pier 28 feet high is 12 ft. × 30 ft. at the base, and 7 ft. × 22 ft. at the top. What will be the cost of the masonry at $12 per cubic yard? (Use the prismoidal formula.) Ans. $3,115.20.

(38) A cast-iron ball which will weigh 25 pounds is required. Figuring cast iron at .26 pound per cubic inch, what will be the diameter of the ball? Ans. 5.68 in., or $5\frac{11}{16}$ in., nearly.

(39) Estimating steel at .283 pound per cubic inch, what will be the weight per foot of the column shown in Fig. 16, in which (*a*) is an enlarged section of a "channel"? Neglect the curved corners and edges on the channels. Ans. 62.83 lb.

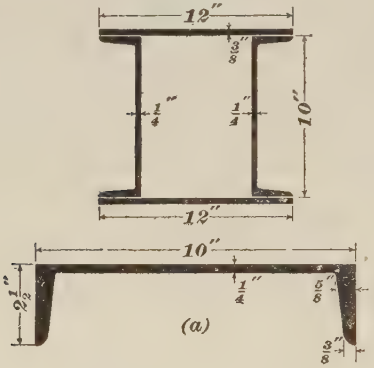

FIG. 16.

(40) The load on a column is 96,000 pounds. Allowing a safe pressure of 3 tons (of 2,000 lb.) per square foot on the soil, and 10 tons per square foot on the brick pier, what must be (*a*) the dimensions of the square footing area, and (*b*) the dimensions of the square column base?

Ans. { (*a*) 4 ft. square. (*b*) 2.19 ft. square.

(41) In a roof truss one of the tie-rods must sustain a load of 27,500 pounds. The safe stress being 10,000 pounds per square inch, what must be the diameter of the rod?

Ans. $1\frac{7}{8}$ in., nearly.

(42) A square pyramidal monument is 3 feet square at the base, 1 foot square at the top, and is 18 feet high. It is capped by a pyramid 1 foot square and 2 feet high. If the stone is granite, weighing 170 pounds per cubic foot, what is the total weight? Ans. $13,373\frac{1}{3}$ lb.

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN THE

QUESTION PAPERS OF THIS VOLUME.

It will be noticed that the Key is divided into sections which correspond to the sections of the Question Papers. The answers and solutions are so numbered as to be similar to the numbers before the questions to which they refer.

ARITHMETIC.

(QUESTIONS 1-75. SEC. 1.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.
605 = Six hundred five.
28,284 = Twenty-eight thousand two hundred eighty-four.
9,006,042 = Nine million six thousand forty-two.
850,317,002 = Eight hundred fifty million three hundred seventeen thousand two.
700,004 = Seven hundred thousand four.

(6) Seven thousand six hundred = 7,600.
Eighty-one thousand four hundred two = 81,402.
Five million four thousand seven = 5,004,007.
One hundred eight million ten thousand one = 108,010,001.
Eighteen million six = 18,000,006.
Thirty thousand ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place

```
  3 2 9 0
    5 0 4
8 6 5 4 0 3
    2 0 7 4
      8 1
        7
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8 7 1 3 5 9   Ans.
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9 units under the units column, and reserve 1 ten for the column of tens. $1+8+7+9 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3 = 11$ thousands, or 1 ten thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten thousand for the column of ten thousands. $1+6 = 7$ ten thousands. Place this 7 ten thousands in the ten thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simple (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0 = 19$; write the 9 and reserve the 1. $1+8+7+0+0+9 = 25$; write the 5 and reserve the 2. $2+0+4+5+2 = 13$; write the 3 and reserve the 1. $1+2+5+3 = 11$; write the 1 and reserve 1. $1+6 = 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{r} 709 \\ 8304725 \\ 391 \\ 100302 \\ 300 \\ 909 \\ \hline 8407336 \end{array}$$ Ans.

(9) (*a*) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin at the *right* to subtract. We cannot subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 ten = 10 units, so we have 10 units + 2 units = 12 units. Then 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5

tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We cannot subtract 3 thousands from 0 thousands, so we take 1 ten thousand from 5 ten thousands and add it to the 0 thousands. 1 ten thousand = 10 thousands, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten thousand from 5 ten thousands and have 4 ten thousands remaining. Since there are no ten thousands in the subtrahend, the 4 in the ten thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r} 50962 \\ 3338 \\ \hline 47624 \end{array}$$ Ans.

(*b*)
$$\begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array}$$ Ans.

(10) (*a*)
$$\begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array}$$ Ans.

(*b*)
$$\begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array}$$ Ans.

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction, we have $\begin{array}{r} 1004 \\ \text{——} \\ \hline 49 \end{array}$ in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl} 1004 & \textit{larger} \\ 955 & \textit{smaller} \\ \hline 1959 & \textit{sum.} \end{array}$$ Ans.

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004) and the difference (49); or, 1,004 − 49 = 955, the smaller

number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals $1,004+955 = 1,959$. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

5962		3874	
8471		2039	
9023		5913	*sum.*
23456	*sum.*		

Subtracting the smaller number (5,913) from the greater (23,456), we have

23456	
5913	
17543	*difference.* Ans.

(13)

\$44675	= amount willed to his son.
26380	= amount willed to his daughter.
\$71055	= amount willed to his two children.
\$125000	= amount willed to his wife and two children.
71055	= amount willed to his two children.
\$53945	= amount willed to his wife. Ans.

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*) 526387
7
3684709 Ans.

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7×3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7×6 thousands = 42 thousands;

$42+2$ thousands reserved $=44$ thousands, or 4 ten thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten thousands. 7×2 ten thousands $=14$ ten thousands; $14+4$ ten thousands reserved $=18$ ten thousands, or 1 hundred thousand and 8 ten thousands. Write the 8 ten thousands and reserve the 1 hundred thousand. 7×5 hundred thousands $=35$ hundred thousands; $35+1$ hundred thousand reserved $=36$ hundred thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. $7\times6=42$; $42+2$ reserved $=44$; write the 4 and reserve 4. $7\times2=14$; $14+4$ reserved $=18$; write the 8 and reserve the 1. $7\times5=35$; $35+1$ reserved $=36$; write the 36.

In this case the multiplier is 17 *units*, or 1 ten and 7 units, so that the product is obtained by adding two partial products, namely, $7\times700{,}298$ and $10\times700{,}298$. The actual operation is performed as follows:

(*b*)

$$\begin{array}{r} 700298 \\ \times\quad 17 \\ \hline 4902086 \\ 700298 \\ \hline 11905066 \end{array}$$ Ans.

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700,298 = 700,298, and write 700,298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*)

```
      2 1 7
      1 0 3
      -----
      6 5 1
    2 1 7 0
    -------
    2 2 3 5 1
          6 7
    ---------
  1 5 6 4 5 7
1 3 4 1 0 6
-----------
1 4 9 7 5 1 7   Ans.
```

Multiply any two of the numbers together and multiply their product by the third number.

(15) If your watch ticks every second, then to find how many times it ticks in 1 week, it is necessary to find the number of seconds in one week.

```
      6 0  seconds = 1 minute.
      6 0  minutes = 1 hour.
    -----
    3 6 0 0  seconds = 1 hour.
      2 4  hours = 1 day.
    -----
  1 4 4 0 0
  7 2 0 0
  -------
  8 6 4 0 0  seconds = 1 day.
          7  days = 1 week.
  -------
6 0 4 8 0 0  seconds in 1 week, or the number of times that
             your watch ticks in 1 week.  Ans.
```

(16) If a monthly publication contains 24 pages, a yearly volume will contain 12×24, or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

```
    2 4
    1 2
    ---
    2 8 8
        8
    -----
  2 3 0 4   Ans.
```

(17) If an engine and boiler are worth $3,246, and the building is worth 3 times as much, plus $1,200, then the building is worth

```
    $ 3 2 4 6
  ×         3
    ---------
      9 7 3 8
+     1 2 0 0
    ---------
  $ 1 0 9 3 8 = value of building.
```

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$$\begin{array}{rl} & \$10938 \\ \times & \underline{\qquad 2} \\ & 21876 \\ + & \underline{\ 1875} \\ & \$23751 = \text{value of tools.} \end{array}$$

$$\begin{array}{rll} \text{Value of building} & = & \$10938 \\ \text{Value of tools} & = & \underline{\ 23751} \\ & & \$34689 = \text{value of the building and tools.} \quad (a) \text{ Ans.} \end{array}$$

$$\begin{array}{rll} \text{Value of engine and boiler} & = & \$\ \ 3246 \\ \text{Value of building and tools} & = & \underline{\ 34689} \\ & & \$37935 = \text{value of the whole plant.} \quad (b) \text{ Ans.} \end{array}$$

(18) (a) $(72\times48\times28\times5)\div(96\times15\times7\times6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72\times48\times28\times5}{96\times15\times7\times6} = ?$$

The 5 in the dividend and 15 in the divisor are both divisible by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. Cross off the 5 and write the 1 over it; also, cross off the 15 and write the 3 under it. Thus,

$$\frac{72\times48\times28\times\overset{1}{\cancel{5}}}{96\times\underset{3}{\cancel{15}}\times7\times6} =$$

The 5 and 15 are not to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and the 15 never existed. Thus,

$$\frac{72\times48\times28\times1}{96\times3\times7\times6} =$$

72 in the dividend and 96 in the divisor are divisible by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. Cross off the 72 and write the 6 over it; also, cross off the 96 and write the 8 under it. Thus,

$$\frac{\overset{6}{\cancel{72}}\times 48\times 28\times 1}{\underset{8}{\cancel{96}}\times 3\times 7\times 6} =$$

The 72 and 96 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 never existed. Thus,

$$\frac{6\times 48\times 28\times 1}{8\times 3\times 7\times 6} =$$

Again, 28 in the dividend and 7 in the divisor are divisible by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. Cross off the 28 and write the 4 over it; also, cross off the 7 and write the 1 under it. Thus,

$$\frac{6\times 48\times \overset{4}{\cancel{28}}\times 1}{8\times 3\times \underset{1}{\cancel{7}}\times 6} =$$

The 28 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 never existed. Thus,

$$\frac{6\times 48\times 4\times 1}{8\times 3\times 1\times 6} =$$

Again, 48 in the dividend and 6 in the divisor are divisible by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. Cross off the 48 and write the 8 over it; also, cross off the 6 and write the 1 under it. Thus,

$$\frac{6\times \overset{8}{\cancel{48}}\times 4\times 1}{8\times 3\times 1\times \underset{1}{\cancel{6}}} =$$

The 48 and 6 are not to be considered any longer, and, in fact, may be erased entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 never existed. Thus,

$$\frac{6\times8\times4\times1}{8\times3\times1\times1} =$$

Again, 6 in the dividend and 3 in the divisor are divisible by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. Cross off the 6 and write the 2 over it; also, cross off the 3 and write the 1 under it. Thus,

$$\frac{\overset{2}{\cancel{6}}\times8\times4\times1}{8\times\underset{1}{\cancel{3}}\times1\times1} =$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 never existed. Thus,

$$\frac{2\times8\times4\times1}{8\times1\times1\times1} =$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2\times\overset{1}{\cancel{8}}\times4\times1}{\underset{1}{\cancel{8}}\times1\times1\times1} = \frac{2\times1\times4\times1}{1\times1\times1\times1}.$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $2\times1\times4\times1 = 8$; the

product of all the uncanceled numbers in the divisor equals $1\times1\times1\times1 = 1$.

Hence, $$\frac{2\times1\times4\times1}{1\times1\times1\times1} = \frac{8}{1} = 8. \quad \text{Ans.}$$

Or, $$\frac{\overset{2}{\cancel{72}}\times\overset{1}{\cancel{48}}\times\overset{4}{\cancel{28}}\times\overset{1}{\cancel{5}}}{\underset{8}{\cancel{96}}\times\underset{3}{\cancel{15}}\times\underset{1}{\cancel{7}}\times\underset{1}{\cancel{6}}} = \frac{8}{1} = 8. \quad \text{Ans.}$$

(b) $(80\times60\times50\times16\times14)\div(70\times50\times24\times20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80\times60\times50\times16\times14}{70\times50\times24\times20} = ?$$

The 50 in the dividend and 70 in the divisor are both divisible by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. Cross off the 50 and write the 5 over it; also, cross off the 70 and write the 7 under it. Thus,

$$\frac{80\times60\times\overset{5}{\cancel{50}}\times16\times14}{\underset{7}{\cancel{70}}\times50\times24\times20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 never existed. Thus,

$$\frac{80\times60\times5\times16\times14}{7\times50\times24\times20} =$$

Also, 80 in the dividend and 20 in the divisor are divisible by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 over it; also, cross off the 20 and write the 1 under it. Thus,

$$\frac{\overset{4}{\cancel{80}}\times60\times5\times16\times14}{7\times50\times24\times\underset{1}{\cancel{20}}} =$$

The 80 and 20 are not to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 never existed Thus,

$$\frac{4\times60\times5\times16\times14}{7\times50\times24\times1} =$$

Again, 16 in the dividend and 24 in the divisor are divisible by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. Cross off the 16 and write the 2 over it; also, cross off the 24 and write the 3 under it. Thus,

$$\frac{4\times60\times5\times\overset{2}{\cancel{16}}\times14}{7\times50\times\underset{3}{\cancel{24}}\times1} =$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 never existed. Thus,

$$\frac{4\times60\times5\times2\times14}{7\times50\times3\times1} =$$

Again, 60 in the dividend and 50 in the divisor are divisible by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. Cross off the 60 and write the 6 over it; also, cross off the 50 and write the 5 under it. Thus,

$$\frac{4\times\overset{6}{\cancel{60}}\times5\times2\times14}{7\times\underset{5}{\cancel{50}}\times3\times1} =$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 never existed. Thus,

$$\frac{4\times6\times5\times2\times14}{7\times5\times3\times1} =$$

The 14 in the dividend and 7 in the divisor are divisible by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1

Cross off the 14 and write the 2 over it; also, cross off the 7 and write the 1 under it. Thus,

$$\frac{4\times6\times5\times2\times\overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}}\times5\times3\times1} =$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 never existed. Thus,

$$\frac{4\times6\times5\times2\times2}{1\times5\times3\times1} =$$

The 5 in the dividend and the 5 in the divisor are divisible by 5, since 5 divided by 5 equals 1. Cross off the 5 of the dividend and write the 1 over it; also, cross off the 5 of the divisor and write the 1 under it. Thus,

$$\frac{4\times6\times\overset{1}{\cancel{5}}\times2\times2}{1\times\underset{1}{\cancel{5}}\times3\times1} =$$

The 5 in the dividend and 5 in the divisor are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 never existed. Thus,

$$\frac{4\times6\times1\times2\times2}{1\times1\times3\times1} =$$

The 6 in the dividend and 3 in the divisor are divisible by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. Cross off the 6 and place the 2 over it; also, cross off the 3 and place the 1 under it. Thus,

$$\frac{4\times\overset{2}{\cancel{6}}\times1\times2\times2}{1\times1\times\underset{1}{\cancel{3}}\times1} =$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in

their stead, and treated as if the 6 and 3 never existed. Thus,

$$\frac{4\times2\times1\times2\times2}{1\times1\times1\times1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

Hence,

$$\frac{\overset{4}{\cancel{8}0}\times\overset{2}{\cancel{6}0}\times\overset{1}{\cancel{5}0}\times\overset{2}{\cancel{1}6}\times\overset{2}{\cancel{1}4}}{\underset{7}{\cancel{7}0}\times\underset{1}{\cancel{5}0}\times\underset{1}{\cancel{2}4}\times\underset{1}{\cancel{2}0}} = \frac{4\times2\times1\times2\times2}{1\times1\times1\times1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

(19) 28 acres of land at \$133 an acre would cost

$$\begin{array}{r} 28\times\$133 = \$3{,}724 \\ 28 \\ \hline 1064 \\ 266 \\ \hline \$3724 \end{array}$$

If a mechanic earns \$1,500 a year and his expenses are \$968 per year, then he would save \$1500 − \$968, or \$532 per year.

$$\begin{array}{r} 968 \\ \hline \$532 \end{array}$$

If he saves \$532 in 1 year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{l} 532)3724(7 \text{ years.} \quad \text{Ans.} \\ \underline{3724} \end{array}$$

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3 × 365 miles) − 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{r} 365 \\ 3 \\ \hline 1095 \\ 246 \\ \hline \textit{difference} \quad 849 \text{ miles.} \quad \text{Ans.} \end{array}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are $354 \times 5{,}280$ feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make $1{,}869{,}120 \div 16$, or 116,820 revolutions.

16)1869120(116820 rev. Ans.
16
26
16
109
96
131
128
32
32

(22) (*a*) 576)589824(1024 Ans.
576
1382
1152
2304
2304

(*b*) 43911)3697306 20(8420 Ans.
351288
184426
175644
87822
87822

(*c*) 505)2527525(5005 Ans.
2525
2525
2525

(*d*) 1234)4961794302(4020903 Ans.

4936

2579

2468

11143

11106

3702

3702

(23) The harness evidently cost the difference between \$444 and the amount which he paid for the horse and wagon. Since \$264 + \$153 = \$417, the amount paid for the horse and wagon, \$444 − \$417 = \$27, the cost of the harness.

\$264 \$444

153 417

\$417 \$27 Ans.

(24) (*a*) (*b*) (*c*)

1024 5005 43911

576 505 8420

6144 25025 878220

7168 250250 175644

5120 2527525 Ans. 351288

589824 Ans. 369730620 Ans.

(25) Since there are 12 months in a year, the number of days the man works is $25 \times 12 = 300$ days. As he works 10 hours each day, the number of hours that he works in one year is $300 \times 10 = 3{,}000$ hours. Hence, he receives for his work $3{,}000 \times 30 = 90{,}000$ cents, or $90{,}000 \div 100 = \$900$. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator, 13, is greater than its denominator, 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.* Ans.

(35) In order to reduce a mixed number to an improper fraction, we must multiply the whole number by the denominator of the fraction and add the numerator of the fraction to that product. This *result* is the numerator of the improper fraction, of which the denominator is the denominator of the fractional part of the mixed number.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\tfrac{5}{16} = \frac{(13 \times 16) + 5}{16} = \tfrac{213}{16}; \quad 10\tfrac{3}{4} = \frac{(10 \times 4) + 3}{4} = \tfrac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator, 13, by the denominator, 2. 2 is contained in 13, 6 times, with 1 remaining. This 1 remaining is written

over the denominator, 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number, 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many ones (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) remaining. Hence, $\frac{13}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number.

$\frac{17}{4} = 4\frac{1}{4}$; $\frac{69}{16} = 4\frac{5}{16}$; $\frac{16}{8} = 2$; $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, invert the divisor (or, in other words, turn it upside down) and then proceed as in multiplication.

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1808}{448} = \frac{452}{112} = \frac{113}{28}$.

$28\overline{)113}\,(4\frac{1}{28}$. Ans.
112
1

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$5\overline{)18}\,(3\frac{3}{5}$. Ans.
15
3

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1$. Ans.

When the denominators of the fractions to be added are alike, we know that the units are divided into the same

number of parts (in this case eighths); we, therefore, add the numerators of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the denominators are *not* alike we know that the units are divided into unequal parts, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to 16ths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4; hence, $\frac{1\times4}{4\times4}=\frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3\times2}{8\times2}=\frac{6}{16}$.

The fractions now have a common denominator, 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{4+6+5}{16}=\frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42+31\frac{5}{8}+9\frac{7}{16}=?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5\times2}{8\times2}=\frac{10}{16}$. Adding the two

fractional parts of the mixed numbers, we have $\frac{10}{16}+\frac{7}{16}$ $=\frac{10+7}{16}=\frac{17}{16}=1\frac{1}{16}$.

The problem now becomes $42+31+9+1\frac{1}{16}=?$

4 2
3 1
9
$1\frac{1}{16}$
8 $3\frac{1}{16}$ Ans.

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

(41) $29\frac{3}{4}+50\frac{5}{8}+41+69\frac{3}{16}=?$ $\frac{3}{4}=\frac{3\times4}{4\times4}=\frac{12}{16}$.

$\frac{5}{8}=\frac{5\times2}{8\times2}=\frac{10}{16}$. $\frac{12}{16}+\frac{10}{16}+\frac{3}{16}=\frac{12+10+3}{16}=\frac{25}{16}=1\frac{9}{16}$

The problem now becomes $29+50+41+69+1\frac{9}{16}=?$

2 9 square inches.
5 0 square inches.
4 1 square inches.
6 9 square inches.
$1\frac{9}{16}$ square inches.
1 9 $0\frac{9}{16}$ square inches. Ans.

(42) (*a*) $\frac{7}{\frac{3}{16}}=7\div\frac{3}{16}=7\times\frac{16}{3}=\frac{7\times16}{3}=\frac{112}{3}=37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that the 7 is to be divided by $\frac{3}{16}$.

(*b*) $\frac{\frac{15}{32}}{\frac{5}{8}}=\frac{15}{32}\div\frac{5}{8}=\frac{15}{32}\times\frac{8}{5}=\frac{\overset{3}{\cancel{15}}\times\overset{1}{\cancel{8}}}{\underset{4}{\cancel{32}}\times\cancel{5}}=\frac{3}{4}$. Ans.

(*c*) $\frac{\frac{4+3}{2+6}}{5}=\frac{\frac{7}{8}}{5}=\frac{7}{8\times5}=\frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32}=\frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8}-\frac{7}{16}=?$ When the denominators of fractions are not alike, it is evident that the units are divided into unequal parts; therefore, before subtracting, reduce the fractions to fractions having a common denominator. Then, subtract the numerators and place the remainder over the common denominator.

$$\frac{7\times2}{8\times2}=\tfrac{14}{16}. \quad \tfrac{14}{16}-\tfrac{7}{16}=\frac{14-7}{16}=\tfrac{7}{16}. \quad \text{Ans.}$$

(*b*) $13-7\frac{7}{16}=?$ This problem may be solved in two ways.

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$$\begin{array}{r} 1\ 2\tfrac{16}{16} \\ 7\tfrac{7}{16} \\ \hline 5\tfrac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$, and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$13=\frac{13}{1}=\frac{13\times16}{1\times16}=\frac{208}{16}$. $7\frac{7}{16}=\frac{(7\times16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}$.

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and $\frac{89}{16}$,

$$\begin{array}{l} =16)\,89\,(\,5\tfrac{9}{16} \\ \quad\ \underline{80} \\ \quad\ \ 9 \end{array}$$

the same result that was obtained by the first method. Ans.

(*c*) $312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16}=\frac{9\times2}{16\times2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$, and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$$\begin{array}{r} 3\ 1\ 2\tfrac{18}{32} \\ 2\ 2\ 9\tfrac{5}{32} \\ \hline 8\ 3\tfrac{13}{32} \end{array}$$

$83+\frac{13}{32}=83\frac{13}{32}$. Ans.

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles. Adding the fractions separately in this case,

$$\tfrac{5}{12}+\tfrac{9}{15}+\tfrac{17}{35}=\tfrac{5}{12}+\tfrac{3}{5}+\tfrac{17}{35}=\frac{175+252+204}{420}=\tfrac{631}{420}=1\tfrac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 8\ 5 \\ 7\ 8 \\ 1\ 2\ 5 \\ 1\tfrac{211}{420} \\ \hline 2\ 8\ 9\tfrac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|l} 5 & 12,\quad 5,\quad 35 \\ \hline 7 & 12,\quad 1,\quad 7 \\ \hline & 12,\quad 1,\quad 1, \text{ or } 5\times7\times12=420. \end{array}$$

(46)

$$\begin{array}{lll} & 5\ 7\ 3\tfrac{4}{5} \text{ tons.} & \tfrac{4}{5}=\tfrac{32}{40} \\ & 2\ 1\ 6\tfrac{5}{8} \text{ tons.} & \tfrac{5}{8}=\tfrac{25}{40} \\ \hline \textit{difference} & 3\ 5\ 7\tfrac{7}{40} \text{ tons. Ans.} & \tfrac{7}{40}=\textit{difference.} \end{array}$$

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans.

(48) Referring to Arts. **114** and **116**,

$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45

$$=\frac{\cancel{2}\times\cancel{3}\times7\times19\times\cancel{11}\times7\times5\times\overset{\overset{3}{\cancel{15}}}{\cancel{45}}}{\cancel{3}\times4\times\cancel{11}\times\underset{4}{\cancel{20}}\times1\times8\times\underset{\cancel{3}}{\cancel{6}}\times1}=\frac{7\times19\times7\times5\times3}{4\times4\times8}=\frac{13,965}{128}$$

$=109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.$ $12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers to improper fractions. $\frac{845}{4}\times\frac{15}{8}=\frac{12,675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently

$$\frac{12,675}{32}\div2\tfrac{1}{2}=\frac{\overset{2,535}{\cancel{12,675}}}{\underset{16}{\cancel{32}}}\times\frac{\cancel{2}}{\cancel{5}}=\frac{2,535}{16}=158\tfrac{7}{16}\text{ pounds.}$$ The

amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

(51)

	tenths.	hundredths.					
.0	8	= *Eight* **hundredths.**					

	tenths.	hundredths.	thousandths.				
.1	3	1	= *One hundred thirty-one* **thousandths.**				

	tenths.	hundredths.	thousandths.	ten-thousandths.			
.0	0	0	1	= *One* **ten-thousandth.**			

	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	0	0	0	2	7	= *Twenty-seven* **millionths.**	

	tenths.	hundredths.	thousandths.	ten-thousandths.			
.0	1	0	8	= *One hundred eight* **ten-thousandths.**			

	tenths.	hundredths.	thousandths.	ten-thousandths.			
93.0	1	0	1	= Ninety-three, and *one hundred one* **ten-thousandths.**			

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths,** so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths,** giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, and one hundred one ten-thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of tenths, hundredths, thousandths, etc. of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the

quotient, or .5, since only one cipher was annexed to the numerator.

$\frac{7}{8}$) 7.000
.875

$\frac{5}{32}$) 5.000000 (.15625 Ans.
32
180
160
200
192
80
64
160
160

Since .65 = $\frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1,000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100}$ = .65. Ans.

$\frac{125}{1000}$ = .125. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator (32.5 + .29 + 1.5) is to be divided by the denominator (4.7 + 9). The operation is as follows:

$$\frac{32.5 + .29 + 1.5}{4.7 + 9} = ?$$

32.5
\+ .29
\+ 1.5

4.7
\+ 9.0
13.7

13.7) 34.290000 (2.5029 Ans.
274
689
685
400
274
1260
1233
27

Since there are 5 decimal places in the dividend, and 1 in the divisor, there are $5-1$ or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283\times\overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283\times\overline{8+5}}{2.63}=?\quad \overline{8+5}=13.$$

$$\begin{array}{r}
1.283\\
\times\quad 13\\
\hline
3849\\
1283\\
\hline
\end{array}$$

$$\begin{array}{r}
2.63\,)\,16.679000\,(\,6.3418\quad\text{Ans.}\\
1578\\
\hline
899\\
789\\
\hline
1100\\
1052\\
\hline
480\\
263\\
\hline
2170\\
2104\\
\hline
66
\end{array}$$

(*c*) $$\frac{\overline{589+27}\times\overline{163-8}}{25+39}=?$$

$$\begin{array}{r}
589\\
+\ 27\\
\hline
616
\end{array}$$

$$\begin{array}{r}
163\\
-\quad 8\\
\hline
155\\
\times 616\\
\hline
930\\
155\\
930\\
\hline
95480
\end{array}$$

```
64)95480.000(1491.875
   64
  314
  256
  588
  576
  120
   64
  560
  512
  480
  448
  320
  320
```

```
 25
+39
 64
```

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

(*d*) $\dfrac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$

```
 406
+ 7.1
 47.7
```

```
 6.27
+ 8.53
 14.80
- 8.01
 6.79
```

```
  3.029
 -1.874
  1.155
 × 47.7
  8085
  8085
 4620
 55.093500(8.1139 Ans
 5432
  773
  679
   945
   679
   2660
   2037
    6230
    6111
     119
```

6 decimal places in the dividend − 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\cancel{8}_{2}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\tfrac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\overset{1}{\cancel{3}}}{16} \times \frac{1}{\cancel{12}_{4}} = \frac{1}{64} \text{ of a foot.}$$

$\frac{1}{64}$) 1.000000 (.015625 Ans.

64

360

320

400

384

160

128

320

320

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, $6 - 0 = 6$ places to be pointed off.

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be $.03617 \times 1{,}500 = 54.255$ lb.

.03617 lb.

1500

1808500

3617

54.255500 lb. Ans.

(62) 72.6 feet of fencing at \$.50 a foot would cost

72.6 × .50, or \$36.30.

.50

\$36.300

If, by selling a carload of coal at a profit of \$1.65 per ton, I make \$36.30, then there must be as many tons

of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

1.65) 36.30 (22 tons. Ans.
330
330
330

(63) 231) 1792.000000 (7.45454, or 77.4545 to four decimal places. Ans.
1617
1722
1617
1050
924
1260
1155
1050
924
1260
1155
1050

(64) $$\frac{37.13 \quad 2 \quad .0952}{\frac{74.26\times24\times3.1416\times19\times19\times350}{\frac{33,000\times12\times4}{1,000 \qquad 2}}}$$

$$= \frac{37.13\times.0952\times19\times19\times350}{1,000} = \frac{446{,}618.947600}{1{,}000}$$

= 446.619 to three decimal places. Ans.

37.13 × .0952 = 3.534776

19 × 19 = 361

361 × 350 = 126350

3.534776 × 126350 = 446618.947600

37.13 × .0952:
7426
18565
33417
3.534776

19 × 19:
171
19
361

361 × 350:
18050
1083
126350

3.534776 × 126350:
17673880 0
10604328
21208656
7069552
3534776
446618.947600

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, place the decimal points directly under each other, and proceed as in the subtraction of whole numbers, placing the decimal point in the remainder directly under the decimal points above.

(*a*)
$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array}$$ Ans.

In the above example we proceed as follows: We cannot subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the 3 hundredths. 1 hundredth = 10 thousandths = 100 ten-thousandths. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 thousandths + 6 ten-thousandths. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths, which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we cannot do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this cannot be done, we take 1 unit from the 9 units. 1 unit = 10 tenths; 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the

tenths place in the remainder. In the minuend we now have 708 units (1 unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*)	(*c*)	(*d*)
81.963	18.00	1.000
1.700	.18	.001
80.263 Ans.	17.82 Ans.	.999 Ans.

(*e*) $872.1-(.8721+.008)=?$ In this problem we are to subtract $(.8721+.008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

.8721
.0080
.8801 *sum.*

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

872.1000
.8801
871.2199 Ans.

(*f*) $(5.028+.0073)-(6.704-2.38)=?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

5.0280
.0073
5.0353 *sum.*

6.704
2.380
4.324 *difference.*

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

5.0353
4.3240
.7113 Ans.

(67) In subtracting a decimal from a fraction, or in subtracting a fraction from a decimal, either reduce the

fraction to a decimal before subtracting or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes $\begin{array}{r} \frac{7}{8}\,)\,\underline{7.000} \\ .875 \end{array}$

$$\begin{array}{r} .875 \\ \underline{.807} \\ .068 \end{array} \quad \text{Ans.}$$

Subtracting .807 from .875, the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction, we have $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$, or, .5. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\begin{array}{r} \frac{3}{8}\,)\,\underline{3.000} \\ .375 \end{array}$ and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .877 \\ \underline{.375} \\ .500 \end{array} \quad \text{Ans.}$$

(*c*) $(\frac{5}{32} + .435) - (\frac{21}{100} - .07) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 60).

Adding .15625 and .435,

$$\begin{array}{rl} & .15625 \\ & \underline{.435} \\ \textit{sum} & .59125 \end{array}$$

$\frac{21}{100} = .21$; subtracting,

$$\begin{array}{rr} & .21 \\ & \underline{.07} \\ \textit{difference} & .14 \end{array}$$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{r} .59125 \\ \underline{.14} \\ .45125 \end{array} \quad \text{Ans.}$$

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths	millionths.		tenths.	hundredths.	thousandths.		
.0	0	0	0	3	3		.5	3			
.0	1	7					.2	7	4		
.0	1	7	0	3	3	*sum.*	.8	0	4	*sum.*	

.8 0 4 *larger sum.*
.0 1 7 0 3 3 *smaller sum.*
difference .7 8 6 9 6 7 Ans.

(68) In addition of decimals the decimal points must be placed directly under each other, so that tenths will come under tenths, hundredths under hundredths, thousandths under thousandths, etc. The addition is then performed as in whole numbers, the decimal point of the sum being placed directly under the decimal points above.

.1 2 5
.7
.0 8 9
.4 0 0 5
.9
.0 0 0 0 2 7
2.2 1 4 5 2 7 Ans.

(69) 9 2 7.4 1 6
8.2 7 4
3 7 2.6
6 2.0 7 9 3 8
1 3 7 0.3 6 9 3 8 Ans.

(70)

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	1	7				
.2						
.0	0	0	0	4	7	
.2	1	7	0	4	7	Ans.

(71) (*a*) There are 3 decimal places in the multiplicand
.1 0 7
.0 1 3
3 2 1
1 0 7
.0 0 1 3 9 1 Ans.
and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

(*b*)

```
      2 0 3
      2.0 3
      -----
      6 0 9
    4 0 6 0
    -------
    4 1 2.0 9
        .2 0 3
    ---------
  1 2 3 6 2 7
8 2 4 1 8 0
-----------
8 3.6 5 4 2 7   Ans.
```

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are $2+0$ or 2 decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are $3+2$ or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

The parenthesis shows that .316 is to be taken from 3.16.

```
3.1 6 0
 .3 1 6
-------
2.8 4 4
```

```
  3 1.8 5
      2.7
---------
2 2 2 9 5
6 3 7 0
---------
8 5.9 9 5
```

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

```
      8 5.9 9 5
        2.8 4 4
    -----------
    3 4 3 9 8 0
  3 4 3 9 8 0
6 8 7 9 6 0
1 7 1 9 9 0
-----------------
2 4 4.5 6 9 7 8 0   Ans.
```

(*d*) $(107.8+6.541-31.96)\times 1.742 = ?$

```
  1 0 7.8
+     6.5 4 1
-----------
  1 1 4.3 4 1
-   3 1.9 6
-----------
    8 2.3 8 1
```

```
    8 2.3 8 1
  × 1.7 4 2
  -----------
  1 6 4 7 6 2
  3 2 9 5 2 4
5 7 6 6 6 7
8 2 3 8 1
-----------------
1 4 3.5 0 7 7 0 2   Ans.
```

(72) (*a*) $(\frac{7}{16}-.13)\times\overline{.625+\frac{5}{8}}=?$

First perform the operation indicated by the parenthesis.

$\frac{7}{16}=\frac{7}{16}$) 7.0 0 0 0 (.4 3 7 5

6 4

6 0

4 8

1 2 0

1 1 2

8 0

8 0

We point off four decimal places, since we annex four ciphers.

.4 3 7 5

.1 3

Subtracting, we obtain .3 0 7 5

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$\frac{5}{8}=\frac{5}{8}$) 5.0 0 0

.6 2 5

Adding the terms included by the vinculum, we obtain

.6 2 5

.6 2 5

1.2 5 0

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

.3 0 7 5

1.2 5

1 5 3 7 5

6 1 5 0

3 0 7 5

.3 8 4 3 7 5 Ans.

(*b*) $(\frac{19}{32}\times.21)-(.02\times\frac{3}{16})=?$

$.21=\frac{21}{100}$. $\frac{19}{32}\times\frac{21}{100}=\frac{399}{3200}$. $.02=\frac{2}{100}$. $\frac{2}{100}\times\frac{3}{16}=\frac{6}{1600}$

$=\frac{3}{800}$. $\frac{3}{800}=\frac{3\times4}{800\times4}=\frac{12}{3200}$. $\frac{399}{3200}-\frac{12}{3200}$

$=\frac{399-12}{3,200}=\frac{387}{3200}$.

Reducing $\frac{387}{3200}$ to a decimal, we obtain

```
 387
3,200 ) 3 8 7.0 0 0 0 0 0 0 (.1 2 0 9 3 7 5   Ans.
        3 2 0 0
        -------
          6 7 0 0
          6 4 0 0
          -------
            3 0 0 0 0
            2 8 8 0 0
            ---------
              1 2 0 0 0
                9 6 0 0
              ---------
                2 4 0 0 0
                2 2 4 0 0
                ---------
                  1 6 0 0 0
                  1 6 0 0 0
                  ---------
```

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $(\frac{13}{4}+.013-2.17)\times\overline{13\frac{1}{4}-7\frac{5}{16}}=?$

$\frac{13}{4}=\frac{13}{4}$

```
4 ) 1 3.0 0
    -------
      3.2 5
```

Point off two decimal places, since two ciphers were annexed to the dividend.

```
   3.2 5
+  .0 1 3
   -------
   3.2 6 3
-  2.1 7
   -------
   1.0 9 3
```

$\frac{5}{16}$ reduced to a decimal is .3125, since

```
 5
16 ) 5.0 0 0 0 (.3 1 2 5
     4 8
     -----
       2 0
       1 6
       -----
         4 0
         3 2
         -----
           8 0
           8 0
           -----
```

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16}=7.3125$, and $13\frac{1}{4}=13.25$, since $\frac{1}{4}=\frac{1}{4}$

```
4 ) 1.0 0
    -------
      .2 5
```

```
   1 3.2 5
-  7.3 1 2 5
   ---------
     5.9 3 7 5
```

```
      5.9 3 7 5
×       1.0 9 3
   -----------
     1 7 8 1 2 5
   5 3 4 3 7 5
 5 9 3 7 5 0
 -----------
 6.4 8 9 6 8 7 5   Ans.
```

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5$ (since $\frac{1}{2} = .5$) $= 1.75$. Ans.

Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then,

$$\frac{7}{8} \div \frac{1}{2} = \frac{7}{\cancel{8}_{4}} \times \frac{\cancel{2}}{1} = \frac{7}{4} = 1\tfrac{3}{4}.$$

Since $\frac{3}{4} = \frac{3}{4}$) 3.0 0 (.7 5, $1\frac{3}{4} = 1.75$, the same answer as above.

2 8

2 0

2 0

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2}$ (since $.5 = \frac{1}{2}$) $= \frac{7}{\cancel{8}_{4}} \times \frac{\cancel{2}}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are 3 decimal places in the dividend and 1 in the divisor, there are $3 - 1$ or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to 16ths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to $\frac{\frac{3}{32}}{\frac{3}{16}} = ?$ $\frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\cancel{3}}{\cancel{32}_{2}} \times \frac{\cancel{16}}{\cancel{3}} = \frac{1}{2}$ or .5. Ans.

(74) $\dfrac{1.25 \times 20 \times 3}{\underline{87 + (11 \times 8)}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

459 + 32

1.2 5

× 2 0

2 5.0 0

× 3

7 5

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87+(11\times 8)}{459+32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87+88=175$.

The numerator is combined as though it were written $87+(11\times 8)$, and its result is

$$\begin{array}{r} 11 \\ \times\ 8 \\ \hline 88 \\ +\ 87 \\ \hline 175 \end{array}$$

The value of the denominator of this fraction is equal to $459+32=491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{75}{1}\div\frac{175}{491} = \frac{75}{1}\times\frac{491}{175} = \frac{\overset{3}{\cancel{75}}\times 491}{\underset{7}{\cancel{175}}} = \frac{1473}{7} = 210\tfrac{3}{7}.$$ Ans.

(75) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And 1.001 − .010001 = .990999. Ans.

ARITHMETIC.

(QUESTIONS 1–92. SEC. 2.)

(1) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. = .25 × 8,428 lb. = 2,107 lb. Ans.

(2) Here \$100 is the base and 1% = .01 is the rate. Then, .01 × \$100 = \$1. Ans.

(3) $\frac{1}{2}$% means one-half of 1 per cent. Since 1% is .01, $\frac{1}{2}$% is .005, for 2).0 1 0. And .005 × \$35,000 = \$175. Ans.
.0 0 5

(4) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **12,**

rate = percentage ÷ base;
rate = 2 ÷ 50 = .04, or 4%. Ans.

(5) By Art. **12,** rate = percentage ÷ base.*
As percentage = 10 and base = 10, we have
rate = 10 ÷ 10 = 1 = 100%.
Hence, 10 is 100% of 10. Ans.

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

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(6) (*a*) Rate = percentage ÷ base. Art. **12.**

As percentage = \$176.54 and base = \$2,522, we have

rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

2 5 2 2) 1 7 6.5 4

.0 7

(*b*) Base = percentage ÷ rate. Art. **11.**

As percentage = 16.96 and rate = 8% = .08, we have

base = 16.96 ÷ .08 = 212. Ans.

.0 8) 1 6.9 6

2 1 2

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **12.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}$%. Ans.

2 1 3.5) 3.2 0 2 5 (.0 1 5 = $1\frac{1}{2}$%.

2 1 3 5

1 0 6 7 5

1 0 6 7 5

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base minus the difference = the percentage. Base = 207 and difference = 201.825; hence, percentage = 207 − 201.825 = 5.175

Rate = percentage ÷ base. Art. **12.**

Therefore, rate = 5.175 ÷ 207 = .025 = .02$\frac{1}{2}$ = $2\frac{1}{2}$%. Ans.

2 0 7) 5.1 7 5 (.0 2 5 = $2\frac{1}{2}$%.

4 1 4

1 0 3 5

1 0 3 5

(7) In this problem \$5,500 is the amount, since it equals what he paid for the farm plus what he gained; 15% is the rate, and the cost (to be found) is the base. Applying rule, Art. **16,**

base = amount ÷ (1 + rate); hence,
base = \$5,500 ÷ (1 + .15) = \$4,782.61. Ans.

```
1.1 5 ) 5 5 0 0.0 0 0 0 ( 4 7 8 2.6 1—
        4 6 0
        -----
          9 0 0
          8 0 5
          -----
            9 5 0
            9 2 0
            -----
              3 0 0
              2 3 0
              -----
                7 0 0
                6 9 0
                -----
                  1 0 0
                  1 1 5
```

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = \$5,500, the selling price.

If 115% = \$5,500, 1% = $\frac{1}{115}$ of \$5,500 = \$47.8261, and 100%, or the cost = 100 × \$47.8261 = \$4,782.61. Ans.

(8) 24 % of \$950 = 950 × .24 = \$228.00
12½% of \$950 = 950 × .125 = 118.75
17 % of \$950 = 950 × .17 = 161.50
53½% of \$950 = \$508.25

The total amount of his yearly expenses, then, is \$508.25; hence, his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, 24% + 12½% + 17% = 53½%, the total percentage of expenditures; hence, 100% − 53½% = 46½% = per cent. saved. And \$950 × .465 = \$441.75 = his yearly savings. Ans.

(9) The percentage is 961.38, and the rate is .37½. By Art. **11**,

base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

.375)961.380000(2563.68
750
2113
1875
2388
2250
1380
1125
2550
2250
3000
3000

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38, and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(10) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **11,**

base = percentage ÷ rate = \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of \$3,750 = \$1,250, and 4 fourths, or the entire property, is $4 \times \$1,250 = \$5,000$. Ans.

(11) Here \$4,810 is the difference and 35% the rate. By Art. **17,**

base = difference ÷ (1 − rate)

= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

.65)4810.00(7400
455
260
260

1.00
.35
.65

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of \$4,810 = \$74, and 100% = $100 \times \$74 = \$7,400$. Ans.

(12) In this example the sales on Monday amounted to \$197.55, which was $12\frac{1}{2}\%$ of the sales for the entire week; i.e., we have given the percentage, \$197.55, and the rate, $12\frac{1}{2}\%$,

and the required number (or the amount of sales for the week) equals the base. By Art. **11,**

base = percentage ÷ rate = \$197.55 ÷ .125;

or,

```
.125 ) 197.5500 ( 1580.4
       125
       ---
        725
        625
        ---
        1005
        1000
        ----
           500
           500
           ---
```

Therefore, base = \$1,580.40, which also equals the sales for the week. Ans.

(13) 16.5 miles = $12\frac{1}{2}\%$ of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, $12\frac{1}{2}\%$ is the rate, and the entire length will be the base. By Art. **11,**

base = percentage ÷ rate = $16.5 \div .12\frac{1}{2}$.

```
.125 ) 16.500 ( 132 miles. Ans.
       125
       ---
        400
        375
        ---
         250
         250
         ---
```

(14) Here we have given the difference, or \$35, and the rate, or 60%, to find the base. We use the rule in Art. **17,**

base = difference ÷ (1 − rate)

= \$35 ÷ (1 − .60) = \$35 ÷ .40 = \$87.50. Ans.

```
.40 ) 35.000 ( 87.5
       320
       ---
        300
        280
        ---
         200
         200
         ---
```

Or, 100% = whole debt; 100% − 60% = 40% = $35.

If 40% = $35, then 1% = $\frac{1}{40}$ of $35 = $\frac{\$35}{40}$, and 100% = $\frac{\$35}{40} \times 100 = \$87.50$. Ans.

(15) 28 rd. 4 yd. 2 ft. 10 in. to inches.

$\times 5\frac{1}{2}$

154

$+ 4$

158 yards.

$\times 3$

474

$+ 2$

476 feet.

$\times 12$

5712

$+ 10$

5722 inches.

Since there are $5\frac{1}{2}$ yards in 1 rod, in 28 rods there are 28 $\times 5\frac{1}{2}$, or 154 yards; 154 yards + 4 yards = 158 yards. There are 3 feet in 1 yard; therefore, in 158 yards there are 3 × 158, or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in 1 foot, and in 476 feet there are 12 × 476, or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(16) 12) 5722 inches.

3) 476 + 10 inches.

$5\frac{1}{2}$) 158 + 2 feet.

28 + 4 yards.

28 rd. 4 yd. 2 ft. 10 in. Ans.

Explanation.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 feet and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are $5\frac{1}{2}$ yards in 1 rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches, there are 28 rd. 4 yd. 2 ft. 10 in.

(17) 5 weeks 3.5 days.

× 7

35 days in 5 weeks.

+ 3.5

38.5 days.

Then, we find how many seconds there are in 38.5 days.

```
    3 8.5 days.
  ×   2 4 hours in 1 day.
  -------
    1 5 4 0
    7 7 0
  -------
    9 2 4.0 hours in 38.5 days.
  ×     6 0 minutes in 1 hour.
  -------
    5 5 4 4 0 minutes in 38.5 days.
  ×       6 0 seconds in 1 minute.
  -----------
  3 3 2 6 4 0 0 seconds in 38.5 days. Ans.
```

(18) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining. Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

```
2 4 ) 1 3 7 5 0 gr.
  2 0 ) 5 7 2 pwt. + 22 gr.
    1 2 ) 2 8 oz. + 12 pwt.
          2 lb. + 4 oz.
```

2 lb. 4 oz. 12 pwt. 22 gr. Ans.

(19)

```
1 0 0 ) 4 7 6 3 2 5 4 li.
  8 0 ) 4 7 6 3 2 + 54 li.
        5 9 5 + 32 ch.
```

595 mi. 32 ch. 54 li. Ans.

EXPLANATION.—There are 100 links in 1 chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in 1 mile; hence, in 47,632 ch. there are as many miles as 80

is contained times in 47,632 ch., or 595 mi. and 32 ch. remaining.

Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(20) 1 7 2 8) 7 6 4 3 2 5 cu. in.

2 7) 4 4 2 + 549 cu. in.

1 6 cu. yd. + 10 cu. ft.

16 cu. yd. 10 cu. ft. 549 cu. in. Ans.

EXPLANATION.—There are 1,728 cu. in. in 1 cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in 1 cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(21) We must arrange the different terms in columns, taking care to have like denominations in the same column.

rd.	yd.	ft.	in.
2	2	2	3
	4	1	9
		2	7
3	$2\frac{1}{2}$	0	7

or, 3 rd. 2 yd. 2 ft. 1 in. Ans.

EXPLANATION.—We begin to add at the right-hand column. 7 + 9 + 3 = 19 in.; as 12 in. make 1 foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) + 2 + 1 + 2 = 6 ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) + 4 + 2 = 8 yd. Since $5\frac{1}{2}$ yd. = 1 rd., 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column and reserve 1 rd. for the next column; 1 (reserved) + 2 = 3 rd.

Ans. = 3 rd. $2\frac{1}{2}$ yd. 0 ft. 7 in.,
or 3 rd. 2 yd. 1 ft. 13 in.,
or 3 rd. 2 yd. 2 ft. 1 in.

(22) We write the compound numbers so that the units of the same denomination shall stand in the same column.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

Beginning to add with the lowest denomination, we find that the sum of the gills is $1+2+3=6$. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) $+5+1+1=8$. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) $+8+3=15$. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) $+4+6+3=16$ gal. Since we cannot reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

(23) Reduce the grains, pennyweights, and ounces to higher denominations.

24)240 gr. 10 pwt. 20)125 pwt. 6 oz. 5 pwt. 12)50 oz. 4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.	
3			
4	2		
	6	5	
		10	
7 lb.	8 oz.	15 pwt.	Ans.

(24) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is $11+29+25+30$

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

+12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107 seconds there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) +17+26+19+16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) +10+20+13+11, or 55 degrees. Since we cannot reduce 55 degrees to any higher denomination, we have 55° 19′ 47″ for the answer.

(25) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11+8+6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column of feet. The sum of the feet is 2 feet (reserved) +2+1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) +4 +5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under

rd.	yd.	ft.	in.
130	5	1	6
215	0	2	8
304	4	0	11
650	$4\frac{1}{2}$	2	1

mi.	rd.	yd.	ft.	in.
or, 2	10	5	0	7 Ans.

the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) + 304 + 215 + 130 = 650 rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard = 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

(26) Since "square links" is the lowest denomination in this problem, we find their sum first, which is 21 + 23 + 16 + 18 + 23 + 21, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is 2 + 3 + 2 + 2 + 2 + 3, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) + 46 + 25 + 56 + 47 + 28 + 21, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

(27) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$, or $\frac{11}{2}$, is contained times in 100 = $18\frac{2}{11}$ rods.

$$100 \div 5\tfrac{1}{2} = 100 \times \tfrac{2}{11} = \frac{100 \times 2}{11} = \frac{200}{11}$$

$$11)\,200\,(\,18\tfrac{2}{11}\text{ rd.}$$

$$\begin{array}{r} 11 \\ \hline 90 \\ 88 \\ \hline 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rod there are $\frac{2}{11} \times \frac{11}{2}$, or 1 yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that the units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2 yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

(28)

A.	sq. rd.	sq. yd.	
114	80	25	
75	70	30	
39	9	$25\frac{1}{4}$	Ans.

Explanation.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We cannot subtract 30 from 25, so we take 1 square rod (= $30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards, leaves $25\frac{1}{4}$ yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from

114 acres, which leaves 39 acres, which we place under the column of acres.

(29) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal $0+1$, or 1 pint. We cannot reduce 1 pint to any higher denomination, so we place it under the pints column. The number of quarts is $3+2$, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quarts column, and reserve the 1 gallon to add to the column of gallons. The number of gallons equals 1 (reserved) $+26+10$, or 37 gallons.

gal.	qt.	pt.
10	2	1
26	3	0
37	1	1

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt., and 1 qt. equals 2 pt.

Beginning with the lowest denomination, we subtract 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint, 1 quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(30) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On Aug. 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19

days cannot be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have $7-1$, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1,850 from 1,892 leaves 42 years. Therefore, he would be 42 years, 1 month, and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(31) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 cannot be subtracted from 3, so we take a unit from the next higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months $+4$ months $=$ 16 months. 7 months from 16 months $=$ 9 months. Since we took 1 year from the years column, we have $1,892-1$, or 1,891 remaining. 1,890 from 1,891 leaves 1 year. Hence, the note ran 1 year, 9 months, and 28 days. Ans.

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

(32) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the

afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(33) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft.	3 in.
	51
879 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $= 867$ ft. 867 ft. $+ 12$ ft. (reserved) $=$ 879 ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3)879 ft. 9 in.

5.5)293 yd.

53 rd. $1\frac{1}{2}$ yd.

Then, product = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in. Ans.

(34)

	qt.	pt.	gi.	
	3	1	3	
			4.7	
	18.2	0	.1	
or,	18 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7×3 gi. = 14.1 gi. As 4 gi. = 1 pt., there are as many pints in 14.1 gi. as 4 is contained times in 14.1 = 3.5 pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7×1 pt. = 4.7 pt.; 4.7 + 3.5 pt. = 8.2 pt. As 2 pt. = 1 qt., there are as many quarts in 8.2 pt. as 2 is contained times in 8.2 = 4.1 qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7×3 qt. = 14.1; 14.1 + 4.1 qt. = 18.2 qt. The answer now is 18.2 qt. 0 pt. .1 gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = $.2 \times 8$ = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

3 qt.
× 2 pt.
6 pt.
+ 1 pt.
7 pt.
× 4 gi.
28 gi.
+ 3 gi.
31 gi.

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

4) 145.7 gi.
2) 36 pt. + 1.7 gi.
18 qt. + 0 pt.

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(35) (3 lb. 10 oz. 13 pwt. 12 gr.) × 1.5 = ?

3 lb. 10 oz. 13 pwt. 12 gr.
× 1 2
3 6 oz.
+ 1 0
4 6 oz.
× 2 0
9 2 0 pwt.
+ 1 3
9 3 3 pwt.
× 2 4
2 2 3 9 2 gr.
+ 1 2
2 2 4 0 4 gr.

22,404 gr. × 1.5 = 33,606 gr.

2 4) 3 3 6 0 6 gr.
2 0) 1 4 0 0 pwt. + 6 gr.
1 2) 7 0 oz. + 0 pwt.
5 lb. + 10 oz.

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pennyweights as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We cannot reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

lb.	oz.	pwt.	gr.	
3	1 0	1 3	1 2	
			1.5	
4.5	1 5	1 9.5	1 8	
or, 4	2 1	1 9	3 0	
or, 5	1 0	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr.; but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the pounds column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(36) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. (reserved) equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu.

bu.	pk.	qt.
2	3	6
		9
1 8	2 7	5 4
or, 2 6	1	6

for the product of the bushels. 9 times 2 bu. plus the 8 bu. (reserved) equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

(37) (7 T. 15 cwt. 10.5 lb.) $\times$ 1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

7 T. 15 cwt. 10.5 lb.
$\times$ 20
140 cwt.
$+$ 15
155 cwt.
$\times$ 100
15500 lb.
$+$ 10.5
15510.5 lb.

15,510.5 lb. $\times$ 1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations. There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 T. and 3 cwt. remaining. Since we cannot reduce 13 T. any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb. $\times$ 16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

100) 26367.85 lb.
20) 263 cwt. + 67.85 lb.
13 T. + 3 cwt.

(38) 7) 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft.
51 A. 31 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. $\times 6$ = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(39)

12)	282 bu.	3 pk.	1 qt.	1 pt.	
	23 bu.	2 pk.	2 qt.	$\frac{1}{4}$ pt.	Ans.

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write 2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. $3 \div 12 = \frac{3}{12}$ or $\frac{1}{4}$ pt.

(40) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mile contains 5,280 ft.; hence 23 mi. contain $5{,}280 \times 23$ = 121,440 ft.

121,440 ft. $\div$ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = $2 \times 4{,}048$, or 8,096 rails. Ans.

(41) In this case, where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either, and then divide as in simple numbers.

1 bu. 1 pk. 7 qt.		3 5 6 bu. 3 pk. 5 qt.	
× 4		× 4	
4 pk.		1 4 2 4 pk.	
+ 1		+ 3	
5 pk.		1 4 2 7 pk.	
× 8		× 8	
4 0 qt.		1 1 4 1 6 qt.	
+ 7		+ 5	
4 7 qt.		1 1 4 2 1 qt.	

```
47)11421(243
   94
   202
   188
    141
    141
```

11,421 qt. ÷ 47 qt. = 243 boxes. Ans.

(42) We must first reduce 16 square miles to acres.

In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16×640 A. = 10,240 A.

```
62)10240 A.
```

1 6 5 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in. Ans.

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are $10 \times 160 = 1{,}600$ sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are $50 \times 30\frac{1}{4}$ sq. yd. = $1{,}512\frac{1}{2}$ sq. yd. 62 is contained in $1{,}512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{4}$ sq. yd. remaining. We write 24 sq. yd. in the quotient and reduce $24\frac{1}{2}$ sq. yd. to sq. ft. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2} \times 144 = 4{,}968$ sq. in.

62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining. We write 80+ sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(43) To square a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11{,}664$. Ans.

$$\begin{array}{r} 108 \\ 108 \\ \hline 864 \\ 108 \\ \hline 11664 \end{array}$$

(44) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5{,}954{,}345.703125$. Ans.

$$\begin{array}{r} 181.25 \\ 181.25 \\ \hline 90625 \\ 36250 \\ 18125 \\ 145000 \\ 18125 \\ \hline 32851.5625 \\ 181.25 \\ \hline 16425781.25 \\ 6570312.50 \\ 3285156.25 \\ 26281250.00 \\ 3285156.25 \\ \hline 59543345.703125 \end{array}$$

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4+2 = 6$ decimal places in the final product.

(45) The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is $27.61 \times 27.61 \times 27.61 \times 27.61 = 581{,}119.73780641$. Ans.

```
                27.61
                27.61
                ----
                2761
             16566
            19327
            5522
            ---------
            762.3121
                27.61
            ---------
            7623121
         45738726
        53361847
       15246242
       ------------
       21047.437081
                27.61
       ------------
       21047437081
    126284622486
   147332059567
   42094874162
   ---------------
   581119.73780641
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4+2 = 6$ decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are $6+2 = 8$ decimal places in the final product.

(46) (*a*) $106^2 = 106 \times 106 = 11{,}236$. Ans.

```
   106
   106
   ---
   636
 1060
 -----
 11236
```

(*b*) $(182\frac{1}{8})^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

```
1/8 = 8 ) 1.000
          .125
```

```
      182.125
      182.125
      -------
      910625
     364250
    182125
   364250
  1457000
  182125
  ------------
  33169.515625
```

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3+3 = 6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

.0 0 5
.0 0 5
.0 0 0 0 2 5

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3+3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

.0 0 6 3
.0 0 6 3
1 8 9
3 7 8
.0 0 0 0 3 9 6 9

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4+4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

1 0.0 6
1 0.0 6
6 0 3 6
1 0 0 6 0 0
1 0 1.2 0 3 6

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2+2 = 4$ decimal places in the product.

(47) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

7 5 3
7 5 3
2 2 5 9
3 7 6 5
5 2 7 1
5 6 7 0 0 9
7 5 3
1 7 0 1 0 2 7
2 8 3 5 0 4 5
3 9 6 9 0 6 3
4 2 6 9 5 7 7 7

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

```
          987.4
          987.4
        -------
         39496
        69118
       78992
      88866
     ----------
     974958.76
          987.4
     ----------
    389983504
   682471132
  779967008
 877462884
 ------------
 962674279.624
```

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1+1=2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 1 in the multiplier, there are $2+1=3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3+3=6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix 4 ciphers to make the 6 decimal places.

```
      .005
      .005
    -------
   .000025
      .005
  ---------
 .000000125
```

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, there are $6+3=9$ decimal places in the final product. In this case we prefix 6 ciphers to form the 9 decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

```
          .4044
          .4044
        -------
         16176
        16176
      161760
     ----------
     .16353936
          .4044
     ----------
      65415744
     65415744
   654157440
  ------------
  .066135317184
```

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4+4=8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8+4=12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(48) $2^5 = 2\times2\times2\times2\times2 = 32$. Ans.

(49) $3^4 = 3\times3\times3\times3 = 81$. Ans.

(50) (*a*) $67.85^2 = 67.85\times67.85 = 4{,}603.6225$. Ans.

$$\begin{array}{r} 67.85 \\ 67.85 \\ \hline 33925 \\ 54280 \\ 47495 \\ 40710 \\ \hline 4603.6225 \end{array}$$

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the product.

(*b*) $967{,}845^2 = 967{,}845\times967{,}845 = 936{,}723{,}944{,}025$. Ans.

$$\begin{array}{r} 967845 \\ 967845 \\ \hline 4839225 \\ 3871380 \\ 7742760 \\ 6774915 \\ 5807070 \\ 8710605 \\ \hline 936723944025 \end{array}$$

(*c*) A fraction may be raised to any power by raising both numerator and denominator to the required term.

Thus, $\left(\frac{3}{8}\right)^2 = \frac{3}{8}\times\frac{3}{8} = \frac{3\times3}{8\times8} = \frac{9}{64}$. Ans.

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4}\times\frac{1}{4} = \frac{1\times1}{4\times4} = \frac{1}{16}$. Ans.

(51) (*a*) $5^{10} = 5\times5\times5\times5\times5\times5\times5\times5\times5\times5 = 9{,}765{,}625$. Ans.

(*b*) $9^5 = 9\times9\times9\times9\times9 = 59{,}049$. Ans.

(*a*)

5
5
25
5
125
5
625
5
3125
5
15625
5
78125
5
390625
5
1953125
5
9765625

(*b*)

9
9
81
9
729
9
6561
9
59049

(52) (*a*) $1.2^4 = 1.2\times1.2\times1.2\times1.2 = 2.0736$. Ans.

1.2
1.2
24
12
1.44
1.2
288
144
1.728
1.2
3456
1728
2.0736

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1+1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2+1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3+1 = 4$ decimal places in the final product.

(*b*) $11^6 = 11\times11\times11\times11\times11\times11 = 1{,}771{,}561$. Ans.

$$\begin{array}{r} 1\,1 \\ 1\,1 \\ \hline 1\,2\,1 \\ 1\,1 \\ \hline 1\,3\,3\,1 \\ 1\,1 \\ \hline 1\,4\,6\,4\,1 \\ 1\,1 \\ \hline 1\,6\,1\,0\,5\,1 \\ 1\,1 \\ \hline 1\,7\,7\,1\,5\,6\,1 \end{array}$$

(*c*) $1^7 = 1\times1\times1\times1\times1\times1\times1 = 1$. Ans.

(*d*) $.01^4 = .01\times.01\times.01\times.01 = .00000001$. Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2+2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

$$\begin{array}{r} .0\,1 \\ .0\,1 \\ \hline .0\,0\,0\,1 \\ .0\,1 \\ \hline .0\,0\,0\,0\,0\,1 \\ .0\,1 \\ \hline .0\,0\,0\,0\,0\,0\,0\,1 \end{array}$$

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4+2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6+2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1\times.1\times.1\times.1\times.1 = .00001$. Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1+1 = 2$ decimal places

.1
.1
.01

in the first product. It is necessary to prefix 1 cipher to the product.

.1
.001

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2+1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

.1
.0001

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3+1 = 4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

.1
.00001

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4+1 = 5$ decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(53) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4+4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix 3 ciphers to form the 8 necessary decimal places in the first product.

.0133
.0133
399
399
133
.00017689
.0133
53067
53067
17689
.000002352637

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8+4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 = 27,273,890.942264331$. Ans.

```
                 301.011
                 301.011
                 ------
                 301011
                301011
             3010110
          9030330
          ----------------
          90607.622121
                 301.011
          ----------------
          90607622121
        90607622121
      906076221210
  2718228663630
  ----------------------
  27273890.942264331
```

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3+3=6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6+3=9$ decimal places in the final product.

(*c*) $(\frac{1}{8})^3 = \frac{1}{8}\times\frac{1}{8}\times\frac{1}{8} = \frac{1\times1\times1}{8\times8\times8} = \frac{1}{512}$. Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$(3\tfrac{3}{4})^3 = \tfrac{15}{4}\times\tfrac{15}{4}\times\tfrac{15}{4} = \frac{15\times15\times15}{4\times4\times4} = \frac{3,375}{64} = 52.734+.$$

Ans.

$$3\tfrac{3}{4} = \frac{3\times4+3}{4} = \frac{12+3}{4} = \tfrac{15}{4}.$$

```
   15        64)3375.000(52.734+.
   15           320
   --           ---
   75            175
  15             128
  ---            ---
  225             470
   15             448
 ----             ---
 1125              220
 225               192
 ----              ---
 3375               280
                    256
                    ---
                     24
```

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(54) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(55) (*a*)

```
  1                √3'4 8'6 7'8 4.4 0'1 0 = 1,867.29+. Ans.
  1                 1
 --                ----
 20                248
  8                224
 --               -----
 28                2467
  8                2196
 ---              ------
 360               27184
   6               26089
 ---              ------
 366          3734 )1095.000 (.293 or .29+
   6                 7468
 ----             -------
 3720              34820
    7              33606
 ----             -------
 3727               12140
    7
 ----
 3734
```

Explanation.—Applying the short method described in Art. **91,** we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1,095, and the last trial divisor (with the cipher omitted) is 3,734. Dividing 1,095 by 3,734, as shown, the quotient, is .293+, or .29+, using two figures. Annexing to the root, gives 1,867.29+. Ans.

```
(b) (a) 3          √9'0 0'0 0'9 9.4 0'0 9'0 0 = 3,000.0165+.
        3     (b) 9                                  Ans
        ---        ------------------
    (d) 60    (c)   0000994009
         0                600001
        ---              ---------
        600              39400800
          0              36000156
        ----             ---------
        6000               3400644
           0
        -----
        60000
            0
        ------
        600000
             1
        -------
        600001
             1
        --------
        6000020
              6
        --------
        6000026
```

Explanation.—Beginning at the decimal point, we point the whole number into periods of two figures each, proceeding from right to left; also, point off the decimal into periods of two figures each, proceeding from left to right. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the trial divisor, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place

another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor, 6,000, is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor, 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor, 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor, 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor, 600,000. 600,000 is contained in 0000994009, once, so we place 1 as the next figure in the root, and also add it to the trial divisor, 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor, 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017—.

In this problem there are seven periods—four in the whole number and three in the decimal—hence, there will be seven figures in the root, four figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017—$.

(*c*)

3	$\sqrt{.0\,0'1\,2'2\,5} = .035.$ Ans.
3	0 0
6 0	1 2
5	9
6 5	3 2 5
	3 2 5

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem solved in Art. **83,** *Arithmetic*, § 2. Since there are three decimal periods in the power, there will be three decimal figures in the root.

(56) (*a*)

1	$\sqrt{1'0\,7'9\,5.2\,1} = 103.9.$ Ans.
1	1
2 0	0 7 9 5
0	6 0 9
2 0 0	1 8 6 2 1
3	1 8 6 2 1
2 0 3	
3	
2 0 6 0	
9	
2 0 6 9	

(*b*)

2	$\sqrt{7'3\,0'0\,8.0\,5} = 270.2+.$ Ans.
2	4
4 0	3 3 0
7	3 2 9
4 7	1 0 8 0 5
7	1 0 8 0 4
5 4 0 0	1
2	
5 4 0 2	

(*c*)

9		81
180		900
4		736
184		16400
4		15104
1880	1896)	1296.00 (.68+ or .7−
8		11376
1888		15840
8		14168
1896		

$\sqrt{9\,0.0\,0\,0\,0\,0\,0} = 9.487-$. Ans.

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.0\,9} = .3$. Ans.

(57)	(*a*)	
6	36	216
6	72	111680
12	10800	98432
6	1504	13248000
180	12304	12650769
8	1568	1424163) 59723100 (.41+ or .4+
188	1387200	5696652
8	18441	2756580
196	1405641	1424163
8	18522	
2040	1424163	
9		
2049		
9		
2058		
9		
2067		

$\sqrt[3]{.3\,2\,7\,6\,8\,0\,0\,0\,0} = .6894+$. Ans.

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **103.**

Explanation.—(1) When extracting the cube root, we divide the power into periods of three figures each. Always

begin at the decimal point and proceed to the left in pointing off the whole number, and to the right in pointing off the decimal. In this power, $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product, 36, at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product, 216, under the figures in the first period. Subtract and bring down the next period, 680, annexing it to the remainder, 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the first correction; multiply the first correction, 12, by the first figure of the root, and we obtain 72 as the product, which added to 36 of column (2) gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the second correction. To this annex one cipher, and add the second figure of the root, and we obtain 188. This multiplied by the second figure of the root, 8, equals 1,504, which added to the trial divisor, 10,800, forms the complete divisor, 12,304. Multiplying the complete divisor, 12,304, by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend, 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period, 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the first new correction. This multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and

annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the new second correction. Dividing the dividend by the trial divisor, 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex one cipher to the new second correction, and to this add the third figure of the root, 9, thereby obtaining 2,049. This multiplied by 9, the third figure of the root, equals 18,441, which added to the trial divisor, 1,387,200, forms the complete divisor, 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

(*b*)

4	16	$\sqrt[3]{74'088} = 42.$ Ans
4	32	64 ·
8	4800	10088
4	244	10088
120	5044	
2		
122		

(*c*)

4	16	$\sqrt[3]{92'416} = 45.212-.$ Ans
4	32	64
8	4800	28416
4	625	27125
120	5425	1291000
5	650	1220408
125	607500	612912) 70592.000 (.115
5	2704	612912
130	610204	930080
5	2708	612912
1350	612912	3171680
2		3064560
1352		107120
2		
1354		

(*d*)

7	49	$\sqrt[3]{.373248} = .72$. Ans.
7	98	343
14	14700	30248
7	424	30248
210	15124	
2		
212		

(58)

1	1	$\sqrt[3]{2.000\,000\,000} = 1.259921+$. Ans.
1	2	1
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4755243) 4383021.000 (.9217 or .922–
2	1850	42797187
360	4687500	10330230
5	33831	9510486
365	4721331	8197440
5	33912	4755243
370	4755243	34421970
5		
3750		
9		
3759		
9		
3768		

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9,217, thus making the last figures of the answer 922, according to Art. **91.** This is not correct in this case, the true answer, to eight decimal places, being 1.25992104+; hence, the first three figures found by

division should be used in this case. The reason for the apparent failure of the method, in this case, to give the seventh figure of the root correctly, is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **91,** if it is desired to obtain absolutely correct results.

(59) (*a*)

1	1	$\sqrt[3]{1'758.416'743} = 12.07.$ Ans.
1	2	1
2	300	758
1	64	728
30	364	30416743
2	68	30416743
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*b*)

1	1	$\sqrt[3]{1'191'016} = 106.$ Ans.
1	2	1
2	30000	191016
1	1836	191016
300	31836	
6		
306		

(*c*) $\sqrt[3]{\frac{4}{32}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}.$ Ans.

(*d*) $\sqrt[3]{\frac{27}{512}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}.$ Ans.

(60)

1	1	
1	2	
2	300	
1	136	
30	436	
4	152	
34	58800	
4	1696	
38	60496	6238092) 1557112.000 (.2496 or .250–
4	1712	12476184
420	6220800	30949360
4	8644	24952368
424	6229444	5996992 0
4	8648	5614282 8
428	6238092	3827092
4		
4320		
2		
4322		
2		
4324		

$\sqrt[3]{3.000000000} = 1.442250-$. Ans.

1
2000
1744
256000
241984
14016000
12458888

(61) (*a*)

1	$\sqrt{1'23.21} = 11.1$. Ans.	
1	1	
20	23	
1	21	
21	221	
1	221	
220		
1		
221		

(*b*)

1	$\sqrt{1'14.92'10} = 10.72+$. Ans.	
1	1	
200	1492	
7	1449	
207	4310	
7	4284	
2140	26	
2		
2142		

(*c*)

7	$\sqrt{50'26'81} = 709$. Ans.	
7	49	
140	1268 1	
0	1268 1	
1400		
9		
1409		

(*d*)

2	$\sqrt{.00'04'12'09} = .0203$. Ans.	
2	00	
400	04	
3	4	
403	1209	
	1209	

(62) (*a*)

1	1	$\sqrt[3]{.006'500'000} = .18663—$. Ans.
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 65144.00 (.627 or .63–
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000} = .2759—$. Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9–
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2	4	$\sqrt[3]{8'036'054'027} = 2{,}003.$ Ans.
2	8	8
4	12000000	036054027
2	18009	36054027
6000	12018009	
3		
6003		

(*d*)

1	1	$\sqrt[3]{.000'004'096} = .016.$ Ans.
1	2	000
2	300	004
1	216	1
30	516	3096
6		3096
36		

(*e*)

2	4	$\sqrt[3]{17.000'000} = 2.5713-.$ Ans.
2	8	8
4	1200	9000
2	325	7625
60	1525	13750000
5	350	13495937
65	187500	198147) 2540700 (.128 or .13–
5	52993	198147
70	192799	559230
5	53488	396294
750	198147	162936
7		
757		
7		
764		

(63) (*a*) In this example, the index is 4, and equals 2×2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ }=\sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6,561}=\sqrt{\sqrt{6,561}}=\sqrt{81}=9$. Ans.

8	$\sqrt{6\ 5'6\ 1}=81$	$\sqrt{81}=9$. Ans.
8	6 4	
1 6 0	1 6 1	
1	1 6 1	
1 6 1		

(*b*) In this example the index is 6, and 6 equals 3×2 or 2×3. The root indicated is the sixth root; hence, extract both the square and the cube roots, it making no particular difference as to which root is extracted first. Thus, $\sqrt[6]{\ }=\sqrt[3]{\ }$ of the $\sqrt{\ }$, or $\sqrt{\ }$ of the $\sqrt[3]{\ }$.

Hence, $\sqrt[6]{117,649}=\sqrt[3]{\sqrt{117,649}}=\sqrt[3]{343}=7$. Ans.

3	$\sqrt{1\ 1'7\ 6'4\ 9}=343$	$\sqrt[3]{3\ 4\ 3}=7$. Ans.
3	9	
6 0	2 7 6	
4	2 5 6	
6 4	2 0 4 9	
4	2 0 4 9	
6 8 0		
3		
6 8 3		

(*c*) $\sqrt[6]{.000064}=\sqrt[3]{\sqrt{.000064}}=.2$. Ans.

$\sqrt{.000064}=.008$. $\sqrt[3]{.008}=.2$. Hence, $\sqrt[6]{.000064}=.2$. Ans.

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since $8\overline{)3.000}$
$.375$

7	49	$\sqrt[3]{.375\,000\,000} = .72112+.$
7	98	343 Ans.
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523) 194639.00 (.124 or .12+
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+.$ Ans.

(64) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}$.

3		$\sqrt{1\,22\,5} = 35$
3		9
60		325
5		325
65		

7	$\sqrt{54\,76} = 74.$
7	49
140	576
4	576
144	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}$. Ans.

(*b*)

```
 5    √.3 3′6 4 = .58.
 5     2 5          Ans.
100     8 6 4
  8     8 6 4
108
```

(*c*)

```
 3    √.1 0′0 0′0 0′0 0 = .31623—.
 3     9                  Ans.
 60     1 0 0
  1      6 1
 61       3 9 0 0
  1       3 7 5 6
 620   632 ) 1 4 4.0 0 (.227 or .23—
   6         1 2 6 4
 626           1 7 6 0
   6           1 2 6 4
 632             4 9 6
```

(*d*) $25.0\frac{3}{4} = 25.075.$

```
 5                 √2 5.0 7′5 0′0 0′0 0′0 0 = 5.00749+.   Ans.
 5                 2 5
10000                 0 7 5 0 0 0
    7                   7 0 0 4 9
10007                     4 9 5 1 0 0
    7                     4 0 0 5 7 6
100140                      9 4 5 2 4 0 0
     4                      9 0 1 3 4 0 1
100144                        4 3 8 9 9 9
     4
1001480
     9
1001489
```

(*e*) $.000\frac{4}{9} = .0004444444+.$

```
 2                 √.0 0′0 4′4 4′4 4′4 4 = .02108+.   Ans.
 2                 0 0
 40                  0 4
  1                    4
 41                      4 4
  1                      4 1
 4200                      3 4 4 4 4
    8                      3 3 6 6 4
 4208                          7 8 0
```

(65) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}$.

$\sqrt{2.0\,0'0\,0'0\,0'0\,0} = 1.41421356+$

1	1
1	1
20	100
4	96
24	400
4	281
280	11900
1	11296
281	60400
1	56564
2820	28284) 3836.0000 (.13562 or .1356+
4	28284
2824	100760
4	84852
28280	159080
2	141420
28282	176600
2	169704
28284	6896

$\sqrt{1.4\,1'4\,2'1\,3'5\,6} = 1.1892+$. Ans.

1	1
1	41
20	21
1	2042
21	1824
1	21813
220	21321
8	49256
228	47564
8	1692
2360	
9	
2369	
9	
23780	
2	
23782	

It is required in this problem to extract the fourth root of 2 to 4 decimal places; hence, we must extract the square root twice, since $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$. In the first operation, we carry the root to 8 decimal places in order to carry the root in the second operation to 4 decimal places.

(*b*) $\sqrt[6]{6} = \sqrt[3]{\sqrt{6}}$.

```
2          √6.00'00'00'00'00 = 2.4494897428+
2           4
40          200
 4          176
44            2400
 4            1936
480            46400
  4            44001
484             239900
  4             195936
4880             4396400
   9             3919104
4889        489896)477296.00000(.974280 or .97428+
   9               4409064
48980               3638960
    4               3429272
48984                2096880
    4                1959584
489880                1372960
     8                 979792
489888                 3931680
     8                 3919168
489896                   12512
```

It is required in this problem to find the sixth root of 6; hence, it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1	1
1	2
2	3 0 0
1	9 9
3 0	3 9 9
3	1 0 8
3 3	5 0 7 0 0
3	1 5 7 6
3 6	5 2 2 7 6
3	1 5 9 2
3 9 0	5 3 8 6 8 0 0
4	3 2 2 2 4
3 9 4	5 4 1 9 0 2 4
4	3 2 2 8 8
3 9 8	5 4 5 1 3 1 2
4	
4 0 2 0	
8	
4 0 2 8	
8	
4 0 3 6	

$\sqrt[3]{2.4\,4\,9'4\,8\,9'7\,4\,2'8\,0\,0} = 1.34801—.$ Ans.

1	
1 4 4 9	
1 1 9 7	
2 5 2 4 8 9	
2 0 9 1 0 4	
4 3 3 8 5 7 4 2	
4 3 3 5 2 1 9 2	
5451312	3 3 5 5 0.0 0 0 (.006 or .01—
	3 2 7 0 7 8 7 2
	8 4 2 1 2 8

(66) (*a*)

1	
1	
2 0	
7	
2 7	
7	
3 4 0	
7	
3 4 7	
7	
3 5 4	

$\sqrt{3.1\,4'1\,6} = 1.7725—.$ Ans.

1	
2 1 4	
1 8 9	
2 5 1 6	
2 4 2 9	
354	8 7.0 0 (.245+ or .25—
	7 0 8
	1 6 2 0
	1 4 1 6
	2 0 4

(*b*)

8
8
160
8
168
8
1760
6
1766
6
1772

$\sqrt{.7\,8'5\,4'0\,0} = .8862+$. Ans.

64
1454
1344
11000
10596
1772) 404.0 (.22 or .2+
3544
496

(67) (*a*)

1
1
2
1
30
4
34
4
38
4
420
6
426
6
432
6
4380
4
4384
4
4388

1
2
300
136
436
152
58800
2556
61356
2592
6394800
17536
6412336
17552
6429888

$\sqrt[3]{3.1\,4\,1'6\,0\,0'0\,0\,0} = 1.4646-$. Ans.

1
2141
1744
397600
368136
29464000
25649344
6429888) 38146560 (.59 or .6–
32149440
5997120

(*b*)

```
  8        64
  8       128
 16      1920000
  8         12025
2400      1932025
    5       12050
2405      1944075
    5
2410
```

```
 ∛.523'600'000 = .80599 + or
   512                .8060—. Ans.
    11600000
     9660125
1944075) 1939875.00 ( .99
         17496675
          19020750
          17496675
```

(68) $11.7 : 13 :: 20 : x.$

$11.7x = 13 \times 20$

$11.7x = 260$

$x = \frac{260}{11.7}$

The product of the means is equal to the product of the extremes.

```
11.7 ) 260.000 ( 22.22+.   Ans.
       234
        260
        234
         260
         234
          260
          234
           26
```

(69) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x.$

$27x = 18 \times 3$

$27x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $(12)^2 : (100)^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144x = 10{,}000 \times 4$

$144x = 40{,}000$

$$x = \frac{40{,}000}{144}$$

```
144 ) 4 0 0 0 0.0 ( 277.7+.   Ans.
      2 8 8
      -----
      1 1 2 0
      1 0 0 8
      -------
        1 1 2 0
        1 0 0 8
        -------
          1 1 2 0
          1 0 0 8
          -------
            1 1 2
```

(70) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \tfrac{84}{7} \text{ or } 12. \quad \text{Ans.}$$

(*b*) In like manner,

$$\frac{x}{24} = \frac{8}{16} \text{ is equivalent to } x : 24 :: 8 : 16.$$
$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \tfrac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \tfrac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x.$
$15x = 45 \times 60$
$15x = 2{,}700$
$x = \frac{2{,}700}{15} = 180.$ Ans.

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600.$
$150x = 10 \times 600$
$150x = 6{,}000$
$x = \frac{6{,}000}{150} = 40.$ Ans.

(71) $x:5::27:12.5.$

$$\begin{array}{r} 5 \\ \hline 12.5\,)\,1\,3\,5.0\,(\,10\tfrac{4}{5}. \quad \text{Ans.} \\ 1\,2\,5 \\ \hline 1\,0\,0 \\ 1\,2\,5 \end{array} = \tfrac{4}{5}.$$

(72) $45:60::x:24.$

$$60x = 45\times 24$$
$$60x = 1{,}080$$
$$x = \frac{1{,}080}{60} = 18. \quad \text{Ans.}$$

(73) $x:35::4:7.$

$$7x = 35\times 4$$
$$7x = 140$$
$$x = \tfrac{140}{7} = 20. \quad \text{Ans.}$$

(74) $9:x::6:24.$

$$6x = 9\times 24$$
$$6x = 216$$
$$x = \tfrac{216}{6} = 36. \quad \text{Ans.}$$

(75)

$\sqrt[3]{1{,}000}:\sqrt[3]{1{,}331}::27:x.$

$10:11::27:x.$

$10x = 297$

$x = \tfrac{297}{10} = 29.7.$ Ans.

$$\begin{array}{r} 1 \\ 1 \\ \hline 2 \\ 1 \\ \hline 3\,0 \\ 1 \\ \hline 3\,1 \end{array}$$

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$\begin{array}{r} 1 \\ 2 \\ \hline 3\,0\,0 \\ 3\,1 \\ \hline 3\,3\,1 \end{array} \qquad \begin{array}{r} 1'3\,3\,1\,(\,11 \\ 1 \\ \hline 3\,3\,1 \\ 3\,3\,1 \\ \hline \end{array}$$

(76) $64:81::21^2:x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64}:\sqrt{81}::\sqrt{21^2}:\sqrt{x^2}$ is the same as

$$8:9::21:x.$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(77) $7+8:7::30:x$ is equivalent to

$$15:7::30:x.$$
$$15x = 7\times 30$$
$$15x = 210$$
$$x = \tfrac{210}{15} = 14. \quad \text{Ans.}$$

(78) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, $29:31 = 2{,}480:x$. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29:31 = x:2{,}480,$$
or $$31x = 71{,}920;$$
whence, $$x = 71{,}920 \div 31 = 2{,}320 \text{ steps.}$$ Ans.

(79) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96:900 = 12:x,$$
or $$96x = 10{,}800;$$
whence, $$x = 10{,}800 \div 96 = 112.5 \text{ mi.}$$ **Ans.**

(80) This is also a direct proportion; hence,

$$27.63:29.4 = .76:x,$$
or $$27.63x = 29.4 \times .76 = 22.344;$$
whence, $$x = 22.344 \div 27.63 = .808+ \text{lb.}$$ Ans.

(81) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt.

Hence, $$23:46 = 5:x,$$
or $$23x = 46 \times 5 = 230;$$
whence, $$x = 230 \div 23 = 10 \text{ days.}$$ Ans.

(82) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2:12^2 = 24:x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2:12^2 = x:24.$$

Dividing both terms of the first couplet (see Art. **129**) by 6,

$$1^2:2^2 = x:24; \text{ or, } 1:4 = x:24;$$
whence, $$4x = 24, \text{ or } x = 6 \text{ degrees.}$$ Ans.

(83) Taking the dimensions as the causes,

$$\begin{array}{|c|c|}\hline 12 & 15 \\ 4 & 5 \\ 2 & 2 \\ 3 & 6 \\ \hline\end{array} = 12 \Big| \; x, \text{ whence, } 2x = 75, \text{ or } x = \$37.50. \quad \text{Ans.}$$

(84) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min. Hence,

$$120 : 868 = 100 : x,$$

or $$120x = 86{,}800;$$

whence, $$x = 723\tfrac{1}{3} \text{ gal.} \quad \text{Ans.}$$

(85) Taking the dimensions as the causes,

$$\begin{array}{|c|c|c|}\hline 14 & 2 & \\ 28 & 20 & \\ 2 & & = 798 \\ 12 & 17 & 399 \\ 10 & 6 & \\ \hline\end{array} \Big| \; x, \text{ whence, } 14x = 17 \times 399 = 6{,}783,$$

or $x = 484\frac{1}{2}$ bbl. Ans.

(86) 8 hr. 40 min. = 520 min. Hence,

$$444 : 1{,}060 = 520 : x,$$

$$\text{or } x = \frac{1{,}060 \times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{137{,}800}{111} = 1{,}241.44+ \text{ min.} = 20 \text{ hr.}$$

41.44+ min. Ans.

(87) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2} : 60 = 6{,}160 : x,$$

or $$x = \frac{60 \times 6{,}160}{5.5} = 67{,}200 \text{ ft.} \quad \text{Ans.}$$

(88) Writing the statement as a direct proportion, $8 : 10 = 5 : x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8 : 10 = x : 5,$$

or $$x = \frac{\overset{4}{\cancel{8}} \times 5}{\underset{2}{\cancel{10}}} = 4 \text{ men.} \quad \text{Ans.}$$

(89) Taking the times as the causes,

$$\begin{array}{r|l|l} 20 & 25 & 14 \\ & 5 & 70 \\ & = 540 & 630; \text{ whence, } 3x = 2\times 14 = 28, \text{ or } x \\ 10 & x \quad 27 & = 9\tfrac{1}{3} \text{ hr. Ans.} \\ 2 & \quad 3 & \end{array}$$

(90) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **168,** 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{r|l|l} & & 9 \\ & & 14 \quad 900 \\ & & 196 \quad 22 \\ 14^2 & 30^2 & 5 \quad 110 \\ 500 & 660 = 112 & x, \text{ or } 500 \quad 660 = 112 \quad x; \\ 48 & 42 & 6 \quad 3 \quad 8 \\ & & 48 \quad 42 \end{array}$$

whence, $x = 9\times 22\times 3 = 594$ horsepower. Ans.

(91) First find the volume of the cylinder in cubic inches, as in the example, Art. **164.** The volume, multiplied by the weight of 1 cubic inch (.261 lb.) will evidently be the weight of the cylinder. Hence,

$$\begin{array}{r|l|l} & & 100 \quad 144 \\ 10^2 & 12^2 & \quad 3 \quad = 1{,}570.8 \quad x; \\ 20 & 60 = 1{,}570.8 \quad x, \text{ or} & \\ & & 20 \quad 60 \end{array}$$

whence, $x = \dfrac{144\times 3\times 1{,}570.8}{100} = 1{,}771.11$ lb. Ans.

(92) Referring to the example in Art. **167,**

$$\begin{array}{r|l|l} & & 5 \quad 4 \\ & & 15 \quad 40 \\ 15 & 40 & 100 \\ 20^2 & 18^2 = 187 & x, \text{ or } 400 \quad 324 = 187 \quad x; \\ 10 & 12 & \quad 4 \\ & & 10 \quad 12 \end{array}$$

whence, $x = \dfrac{324\times 4\times 187}{500} = 484.7$ lb. Ans.

FORMULAS.

(QUESTIONS 1–9. SEC. 3.)

(1) Substituting for D, x, B, and i their values,

$$C = \frac{D-x}{B+i} = \frac{120-12}{10+3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(2) Substituting for A, h, D, and x their values,

$$Q = \frac{A\,h+D}{2x+6} + D = \frac{(5\times 200)+120}{(2\times 12)+6} = \frac{1{,}000+120}{24+6}$$

$$= \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(3) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246\times B\times h}{\dfrac{A\,x+h}{A\,i-B}} = \frac{3.246\times 10\times 200}{\dfrac{(5\times 12)+200}{(5\times 3.5)-10}} = \frac{6{,}492}{\dfrac{260}{7.5}}$$

$$= 6{,}492 \div \frac{260}{7.5} = 6{,}492\times\frac{7.5}{260} = 187.269+. \quad \text{Ans.}$$

(4) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B+1.5}} = \sqrt{\frac{5\times 120}{(3.5\times 10)+1.5}} = \sqrt{\frac{600}{36.5}}$$

$$= \sqrt{16.4383} = 4.05+. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(5) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{B\,x}{.00018\,h\,(A^2-x)}} = \sqrt[3]{\frac{10\times 12}{.00018\times 200\times(5^2-12)}}$$

$$= \sqrt[3]{\frac{120}{.036\times(25-12)}} = \sqrt[3]{\frac{120}{.036\times 13}} = \sqrt[3]{\frac{120}{.468}}$$

$$= \sqrt[3]{256.41+} = 6.35+. \quad \text{Ans.}$$

(6) Substituting for h, D, and A their values,

$$f = \frac{10\,(h-D)^2}{\sqrt[3]{D+A}} = \frac{10\,(200-120)^2}{\sqrt[3]{120+5}} = \frac{10\times 80^2}{\sqrt[3]{125}} = \frac{64{,}000}{5}$$

$$= 12{,}800. \quad \text{Ans.}$$

(7) Substituting for B, A, and D their values,

$$g = \frac{(B-A)^2-\sqrt[3]{D+A}}{A^3-(1+D)} = \frac{(10-5)^2-\sqrt[3]{120+5}}{5^3-(1+120)} = \frac{5^2-\sqrt[3]{125}}{125-121}$$

$$= \frac{25-5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(8) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}} = \sqrt{\frac{5\times 10^2}{\sqrt[3]{5\times 200}}} = \sqrt{\frac{5\times 100}{\sqrt[3]{1{,}000}}} = \sqrt{\frac{500}{10}}$$

$$= \sqrt{50} = 7.071+. \quad \text{Ans.}$$

(9) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490+\frac{(h\,x)^2}{D^2}\right]}{h+\frac{x}{D}\,(A^2-B)^2}} = \sqrt{\frac{5^2\left[490+\frac{(200\times 12)^2}{120^2}\right]}{200+\frac{12}{120}\,(5^2-10)^2}}$$

$$= \sqrt{\frac{25\,(490+400)}{200+(\frac{1}{10}\times 225)}} = \sqrt{\frac{25\times 890}{200+22.5}} = \sqrt{\frac{22{,}250}{222.5}}$$

$$= \sqrt{100} = 10. \quad \text{Ans.}$$

GEOMETRY AND MENSURATION.

(QUESTIONS 1–42. SEC. 4.)

(1) (*a*) The angles of the triangles being equal, the bevel angle is 60°. Ans.

(*b*) $CE = 2.5$ ft. $OG = \sqrt{(OC)^2-(GC)^2} = \sqrt{(2.5)^2-(1.25)^2}$ $= \sqrt{4.69} = 2.16$ ft. $DF : CE = OH : OG$; or, $DF = \frac{CE \times OH}{OG} = \frac{2.5 \times 2.5}{2.16} = 2.89$ ft. $= 2$ ft. 10.7 in. Ans.

(*c*) Since the width, as IH, is 8 in. $= .67$ ft., $OI = 2.5 - .67 = 1.83$ ft. Then, $AB : DF = OI : OH$; or, $AB = \frac{DF \times OI}{OH} = \frac{2.89 \times 1.83}{2.5} = 2.12$ ft. $= 2$ ft. $1\frac{1}{2}$ in., nearly. Ans.

(2) By drawing a diagram, it will be seen that $\frac{1}{2}$ of 12 ft. $= 6$ ft. is the longer side of a right triangle, whose hypotenuse is $\frac{1}{2}$ of 13 ft. $= 6.5$ ft. Then the shorter side of the triangle $= \sqrt{(6.5)^2-6^2} = 2.5$ ft., which is one-half a side of the octagon. Hence, the entire perimeter of the octagon is $2.5 \times 2 \times 8 = 40$ ft. Ans.

(3) (*a*) $CD = OD - OC$. $OC = \sqrt{(AO)^2-(AC)^2}$ $= \sqrt{3^2-(2.25)^2} = 1.98$ ft. $OD = 3$ ft.; hence, $CD = 3 - 1.98 = 1.02$ ft. $= 1$ ft. $\frac{1}{4}$ in. Ans.

(*b*) $OC = OD - CD$; $AB = 2AC = 2\sqrt{(OA)^2-(OC)^2}$. Ans.

thickness being $1\frac{1}{4}$ in., the volume is $3.43 \times 1.25 = 4.29$ cu. in., and at .26 lb. per cu. in., the weight is 1.11 lb. Ans.

(16) 30 in. = 2.5 ft.; 18 in. = 1.5 ft. Area of lower base is $2.5 \times 2.5 = 6.25$ sq. ft.; of upper base is $1.5 \times 1.5 = 2.25$ sq. ft. By the formula for frustum of pyramid, the volume is

$$\frac{h}{3}(A + a + \sqrt{A \times a}) = \tfrac{6}{3}(6.25 + 2.25 + \sqrt{6.25 \times 2.25})$$

$= 2(8.50 + 3.75) = 24.5$ cu. ft. Or, in the prismoidal formula, the mean area being $\left(\frac{2.5 + 1.5}{2}\right)^2 = 4$ sq. ft., the volume is $\frac{h}{6}(A + a + 4M) = \frac{6}{6}(6.25 + 2.25 + 16) = 24.5$ cu. ft. As there are $22\frac{1}{2}$ brick to a cu. ft., each pier will contain $22.5 \times 24.5 = 551.25$ brick; 18 piers will therefore require $551.25 \times 18 = 9{,}922.5$ brick. Ans.

(17) The area of the ring is the difference between a 7-inch and a 6-inch circle, or $\frac{\pi}{4}(7^2 - 6^2) = 10.21$ sq. in.; that of the flanges $= \frac{3}{4}$ in. $\times 1$ in. $\times 4 = 3$ sq. in., making a total of 13.21 sq. in. Since a strip 1 foot long and 1 inch square weighs 3.33 lb., 1 foot of the column will weigh $13.21 \times 3.33 = 43.99$ lb. Increasing this by 2 per cent., the weight per foot is $43.99 \times 1.02 = 44.87$ lb. Ans.

(18) Reducing feet and inches to feet and decimals:

1 piece,	$4.5 \times 3.75 \times 1.25$	$=$	2 1.0 9 cu. ft.
1 piece,	$4.25 \times 3.5 \times 1.25$	$=$	1 8.5 9 cu. ft.
1 piece,	$5.25 \times 4.0 \times 1.25$	$=$	2 6.2 5 cu. ft.
3 pieces,	$3.25 \times 3.5 \times 1.25$	$=$	4 2.6 6 cu. ft.
2 pieces,	$5.5 \times 4.0 \times 1.25$	$=$	5 5.0 0 cu. ft.
1 piece,	$5.0 \times 3.75 \times 1.25$	$=$	2 3.4 4 cu. ft.
1 piece,	$4.75 \times 4.5 \times 1.25$	$=$	2 6.7 2 cu. ft.
2 pieces,	$4.5 \times 4.25 \times 1.25$	$=$	4 7.8 2 cu. ft.
Total,			2 6 1.5 7 cu. ft.

261.57 cu. ft., at 140 lb. per cu. ft., weigh 36,619.8 lb. = 366.2 hundred pounds. At 21 cents per hundred, the freight charges are $366.2 \times \$.21 = \$76.90$. Ans.

(19) (*a*) MD, DL, KB, BI, AH, and AG are all equal, and their true length is the hypotenuse, as $M'D'$, of a right triangle, the sides of which, as $M'D''$ and $D'D''$, are each 10 ft.; hence, $M'D'$, etc. $= \sqrt{10^2+10^2} = 14.14$ ft.

Area, $ABIH = H'A' \times HI = 14.14 \times 36 =$ 5 0 9.0 4 sq.ft.

Area, $GFCA = \frac{1}{2}(GF+AC) \times A'G'$

$= \frac{12+22}{2} \times 14.14 =$ 2 4 0.3 8 sq.ft.

Area, $EKBC = \frac{1}{2}(EK+CB) \times K'B'$

$= \frac{4+14}{2} \times 14.14 =$ 1 2 7.2 6 sq.ft.

Area, $MDCF$ and $DLEC$

$= 2\left[\frac{1}{2}(MF+DC) \times M'D'\right]$

$= 2\left(\frac{10+20}{2} \times 14.14\right) =$ 4 2 4.2 0 sq.ft.

Total, 1,3 0 0.8 8 sq.ft.

Ans.

(*b*) The length of the valley FC is the hypotenuse of a right triangle *on the roof surface* of which the real length of FN (which is $M'D'$) and NC ($= AG = 10$ ft.) are the sides; hence, $FC = \sqrt{(M'D')^2+(NC)^2} = \sqrt{(14.14)^2+10^2}$ $= \sqrt{300} = 17.32$ ft. $= 17$ ft. $3\frac{7}{8}$ in. Ans.

(20) The quantity discharged is the volume of a cylinder whose length is $5 \times 60 = 300$ ft.; the area of a 4-inch circle is 12.57 sq. in. $= .09$ sq. ft.; hence, the volume is $.09 \times 300$ $= 27$ cu. ft., which, reduced to gallons, gives 27×7.5 $= 202.5$ gal. Ans.

(21) The volume of the metal will be $3 \div .41 = 7.32$ cu. in. The area of a $1\frac{3}{4}$-inch circle is 2.4 sq. in.; that of a $1\frac{1}{2}$-inch circle $= 1.77$ sq. in.; the area of the ring is, therefore, $2.40 - 1.77 = 0.63$ sq. in.; hence, the length required is $7.32 \div 0.63 = 11.6$ in. Ans.

(22) (*a*) In the formula

$$r = \frac{c^2+4h^2}{8h},$$

$c = 6$ ft. 8 in. $= 80$ in., and $h = 8$ in. Substituting,

$$r = \frac{80^2+4\times8^2}{8\times8} = 104 \text{ in.} = 8 \text{ ft. } 8 \text{ in.} \quad \text{Ans.}$$

(*b*) In the formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3},$$

$c = 80$ in., $h = 8$ in. Substituting,

$$l = \frac{4\sqrt{6{,}400 + 256} - 80}{3} = \frac{4 \times 81.58 - 80}{3} = 82.11 \text{ in.},$$

the length of arc; as there are 13 stones, the length of each is $82.11 \div 13 = 6.32$ in. Ans.

(23) Since AC is a mean proportional to the height CD and the other part of the diameter, which we will denote by S; then,

$$S \times CD = \overline{AC}^2; \text{ or, } S = \frac{\overline{AC}^2}{CD} = \frac{40^2}{8} = 200 \text{ in.},$$

The diameter is the sum of the two parts, or $S + CD = 200 + 8 = 208$ in. The radius is, therefore, $208 \div 2 = 104$ in., or 8 ft. 8 in. Ans.

(24) The area required is the difference in areas of a 12-inch circle, and the segment ADB. The area of the circle is $.7854 \times 12^2 = 113.10$ sq. in. The length $AC = \sqrt{(AO)^2 - (OC)^2} = \sqrt{6^2 - 3^2} = 5.2$ in., and $AB = 5.2 \times 2 = 10.4$ in. The length of the arc ADB is found by formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}.$$

$c = AB = 10.4$ in.; $h = CD = 3$ in.; hence,

$$l = \frac{4\sqrt{108.16 + 36} - 10.4}{3} = \frac{4 \times 12.0 - 10.4}{3} = 12.53 \text{ in}$$

The area of sector

$$OADB = \frac{lr}{2} = \frac{12.53 \times 6}{2} = 37.59 \text{ sq. in.};$$

the area of triangle is $\frac{10.4 \times 3}{2} = 15.60$ sq. in., whence, area of segment $ADB = 37.59 - 15.60 = 21.99$ sq. in. Deducting 21.99 sq. in. from 113.10 sq. in., the area of the circle, the remainder is 91.11 sq. in. Ans.

(25)

3 pieces, 3″×10″×16′ 0″	=	1 2 0	ft. B. M.	
8 pieces, 3″×10″×12′ 6″	=	2 5 0	ft. B. M.	
6 pieces, 3″× 8″×14′ 6″	=	1 7 4	ft. B. M.	
10 pieces, 3″× 8″× 9′ 0″	=	1 8 0	ft. B. M.	
4 pieces, 3″× 6″×10′ 6″	=	6 3	ft. B. M.	
20 pieces, 3″× 6″× 7′ 0″	=	2 1 0	ft. B. M.	
8 pieces, 4″× 6″×18′ 0″	=	2 8 8	ft. B. M.	
4 pieces, 4″× 6″× 6′ 6″	=	5 2	ft. B. M.	
52 pieces, 2″× 4″×18′ 0″	=	6 2 4	ft. B. M.	
15 pieces, 2″× 4″× 9′ 6″	=	9 5	ft. B. M.	
600 ft. B. M. 1″×6″ flooring	=	6 0 0	ft. B. M.	
200 ft. B. M. 2-inch plank	=	2 0 0	ft. B. M.	
Total,		2,8 5 6	ft. B. M.	Ans.

(26) Perimeter of the house is $12+10+12+14+24+24 = 96$ ft. The height of it being 22 ft., the area is $96\times22 = 2{,}112$ sq. ft. Adding for gables, one $\frac{14\times7}{2} = 49$ sq. ft., and one $\frac{12\times7}{2} = 42$ sq. ft., the total area is $2{,}112+49+42 = 2{,}203$ sq. ft. Deduct 10 windows, 3 ft. $\times$ 7 ft. $= 210$ sq. ft.; 10 windows, 3 ft. $\times$ 6 ft. 0 in. $= 180$ sq. ft.; and 4 doors, 3 ft. 6 in. $\times$ 8 ft. $= 112$ sq. ft. The total deduction is $210+180+112 = 502$ sq. ft.; therefore, the net surface is $2{,}203-502 = 1{,}701$ sq. ft., requiring, at 7 bricks per sq. ft., $1{,}701\times7 = 11{,}907$ bricks. Ans.

(27) The major axis of the glass is 8 ft. $-$ (4 in. $\times$ 2) $= 7$ ft. 4 in. $= 7.33$ ft.; the minor axis is 5 ft. $-$ (4 in. $\times$ 2) $= 4$ ft. 4 in. $= 4.33$ ft. Then the area is, by the formula $A = .7854dD = .7854\times4.33\times7.33 = 24.93$ sq. ft. Ans.

(28) The area of the semielliptical arch is one-half of $.7854\,dD$; $d = 4\times2 = 8$ ft.; $D = 10$ ft.; hence, the area is $\frac{1}{2}(.7854\times8\times10) = 31.42$ sq. ft. Adding this to the rectangular area, 6 ft. $\times$ 10 ft. $= 60$ sq. ft., the total area is 91.42 sq. ft., which exceeds the required area, 88 sq. ft., by 3.42 sq. ft. **Ans.**

(29) Find the area of cross-section, beginning at the bottom:

	7.5 ft. × 1 ft. =	7.5 0 sq. ft.
	6.5 ft. × 1 ft. =	6.5 0 sq. ft.
Triangular portion,	$\frac{1.25 \text{ ft.} \times 15 \text{ ft.}}{2}$ =	9.3 7 sq. ft.
Center part,	2 ft. × 15 ft. =	3 0.0 0 sq. ft.
Under "steps"—		
1 strip, 3.7 5		
1 strip (3.75 × 2), 7.5 0	.75 ft. × 22.5 =	1 6.8 7 sq. ft.
1 strip (3.75 × 3), 1 1.2 5		
Total, 2 2.5 0		
Coping,	2.5 ft. × .5 ft. =	1.2 5 sq. ft.
Total area of cross-sections,		7 1.4 9 sq. ft.

The length being 16 ft., the contents are 71.49 × 16 = 1,143.84 cu. ft. = 1,143.84 ÷ 27 = 42.36 cu. yd. Ans.

(30) In the formula

$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3},$$

$c = 3\frac{1}{2}$ ft. = 42 in.; $h = 3$ in.; substituting,

$$l = \frac{4\sqrt{42^2 + 3^2 \times 4} - 42}{3} = \frac{4\sqrt{1{,}800} - 42}{3} = 42.57 \text{ in.} = 42\tfrac{9}{16} \text{ in.},$$

or 3 ft. $6\frac{9}{16}$ in. Ans.

(31) $AF + 2$ ft. = 26 ft.; $AB + 2$ ft. = 20 ft.

$AF \times AB$ = 5 2 0 sq. ft.

$CD = 12$ ft.; $DE + 2$ ft. = 16 ft.

$CD \times DE$ = 1 9 2 sq. ft.

Total, 7 1 2 sq. ft.

712 × 9 = 6,408 cu. ft., and 6,408 ÷ 27 = 237.33 cu. yd. Ans.

(32) (*a*) Since the height of the gables is $\frac{5}{8}$ the width, 24 ft., it is $\frac{5}{8} \times 24 = 15$ ft. The length of the roof slope is the hypotenuse of a right triangle, the sides of which are 15 ft. and 12 ft. (one-half the width); it is, therefore, $\sqrt{15^2 + 12^2} = 19.21$ ft. Adding 15 in. = 1.25 ft., the total width of each half of the roof is 19.21 + 1.25 = 20.46 ft. The length

is 32 ft. + (15 in. × 2) = 34.5 ft. The area of each half is 20.46 × 34.5 = 705.87 sq. ft.; the area of the whole is, therefore, 705.87 sq. ft. × 2 = 1,411.74 sq. ft., which is equal to 1,411.74 ÷ 100 = 14.12 squares. Ans.

(*b*) The exposed area of a slate is $12 \times 8\frac{1}{2} = 102$ sq. in. In a square there are 144 × 100 = 14,400 sq. in. Since 1 slate covers 102 sq. in., to cover 14,400 sq. in. will require 14,400 ÷ 102 = 141+ slates. Ans.

(33) In the formula for surface of a sphere, $S = \pi d^2$, d is 3 ft.; the surface of the hemisphere is, therefore, $\frac{1}{2}\pi d^2 = \frac{1}{2} \times 3.1416 \times 9 = 14.14$ sq. ft. Deducting 5 sq. ft. for openings, the net surface is 9.14 sq. ft. Ans.

(34) The outer diameter is 10 in. + ($\frac{1}{2}$ in. × 2) = 11 in. The area of an 11-inch circle is 95.03 sq. in.; that of a 10-inch circle is 78.54 sq. in.; the area of the ring is, therefore, 95.03 − 78.54 = 16.49 sq. in. The length is 12 ft. = 144 in.; multiplying 16.49 by 144, the contents are 2,374.56 cu. in. = 2,374.56 ÷ 1,728 = 1.37 cu. ft.; 1.37 cu. ft., at 450 lb. per cu. ft., weigh 1.37 × 450 = 616.5 lb. Ans.

(35) (*a*) The circumference of the tower = πd = 3.1416 × 8 ft. = 25.13 ft. The area of the surface is 25.13 × 28 (the height) = 703.64 sq. ft., which divided by 100, the number of sq. ft. in a square, gives 7.04 squares. Ans.

(*b*) The exposed area of a shingle is 5 × 6 = 30 sq. in. There are 144 sq. in. × 100 = 14,400 sq. in. in a square, which will therefore require 14,400 ÷ 30 = 480 shingles. Ans.

(36) The area of the base plate is 15 × 15 = 225 sq. in., less the area of a 7-inch circle and the portion cut out at the corners, which is equal (for four corners) to the difference between a 4-inch square and a circle 4 inches in diameter; or, 16 sq. in. − 12.57 sq. in. = 3.43 sq. in. The area of a 7-inch circle is 38.48 sq. in. Adding 38.48 and 3.43, the total deduction is 41.91 sq. in., which subtracted from 225 sq. in. leaves 183.09 sq. in. The thickness being $1\frac{1}{2}$ in. = 1.5 in., the contents are 183.09 × 1.5 = 274.63 cu. in.

The height of the cylindrical part is 2 in.; its area is the difference between the areas of an 11-inch and a 10-inch circle, which is $95.03 - 78.54 = 16.49$ sq. in.; hence, the contents are $16.49 \times 2 = 32.98$ cu. in. The triangular side area of the ribs is $\frac{2+0}{2} \times 4\frac{1}{2}$ in. $= 4.5$ in.; the thickness being $\frac{1}{2}$ in., the contents of each are $4.5 \times .5 = 2.25$ cu. in.; and for four the contents are $2.25 \times 4 = 9$ cu. in.

Adding the several portions, the total contents $= 274.63 + 32.98 + 9 = 316.61$ cu. in. The weight is, therefore, $316.61 \times .26 = 82.32$ lb. Ans.

(37) The area of the lower base is $12 \times 30 = 360$ sq. ft.; that of the upper base is $7 \times 22 = 154$ sq. ft. The area of a mid-section is

$$\frac{12+7}{2} \times \frac{30+22}{2} = 9.5 \times 26 = 247 \text{ sq. ft.}$$

By the prismoidal formula, the volume is

$$\tfrac{2}{6}\tfrac{8}{}(360 + 154 + 247 \times 4) = \tfrac{2}{6}\tfrac{8}{} \times 1{,}502 = 7{,}009.33 \text{ cu. ft.}$$
$$= 7{,}009.33 \div 27 = 259.6 \text{ cu. yd.}$$

At \$12.00 per cu. yd., the cost is $259.6 \times 12 = \$3{,}115.20$. Ans.

(38) The volume of the ball is $25 \div .26 = 96.15$ cu. in. The diameter $= \sqrt[3]{\frac{96.15}{.5236}} = \sqrt[3]{183.63} = 5.68$ in. $= 5\frac{11}{16}$ in., nearly. Ans.

(39) The area of the two plates is $2(12 \times \frac{3}{8}) = 9$ sq. in. Each channel consists of a rectangle which has an area of $10 \times \frac{1}{4} = 2\frac{1}{2}$ sq. in., and 2 trapezoids each ($2\frac{1}{2}$ in. $- \frac{1}{4}$ in.) long, and with an average width of $\frac{\frac{5}{8}+\frac{3}{8}}{2} = \frac{1}{2}$ in., and an area of $2 \times (2\frac{1}{4} \times \frac{1}{2}) = 2\frac{1}{4}$ sq. in., making a total for each channel of $2\frac{1}{2} + 2\frac{1}{4} = 4\frac{3}{4}$ sq. in.; for two channels the area is $4\frac{3}{4} \times 2 = 9\frac{1}{2}$ sq. in. Adding the area of the plates, the total is $9\frac{1}{2} + 9 = 18\frac{1}{2}$ sq. in. Multiplying by 12 (inches in a foot), the contents per foot of length are $18\frac{1}{2} \times 12 = 222$ cu. in., and the weight at .283 lb. per cu. in. is $222 \times .283 = 62.83$ lb. Ans.

(40) (*a*) The area of the footing is $\frac{96{,}000}{2{,}000\times 3} = 16$ sq. ft. Each side is therefore $\sqrt{16}$ ft. = 4 ft. Ans.

(*b*) The area of the base plate is $\frac{96{,}000}{2{,}000\times 10} = 4.8$ sq. ft.; and each side is $\sqrt{4.8}$ ft. = 2.19 ft. Ans.

(41) The required area is $\frac{275000}{100000} = 2.75$ sq. in. The diameter $= \sqrt{\frac{2.75}{.7854}} = 1.87$ in. $= 1\frac{7}{8}$ in., nearly.

(42) The monument consists of a frustum of a pyramid and a pyramid. For the former, the area of the lower base is $3\times 3 = 9$ sq. ft.; that of the upper base is $1\times 1 = 1$ sq. ft. Using the formula

$$V = \frac{h}{3}(A + a + \sqrt{A\times a}) = \tfrac{1}{3}\tfrac{8}{3}(9 + 1 + \sqrt{9\times 1}) = 78 \text{ cu. ft.}$$

The volume of the pyramid is $\frac{a\,h}{3}$; $a = 1$ sq. ft.; $h = 2$ ft.; therefore, $\frac{a\,h}{3} = \frac{1\times 2}{3} = \frac{2}{3}$ cu. ft. The total is $78 + \frac{2}{3} = 78\frac{2}{3}$ cu. ft., and the weight at 170 lb. per cu. ft., is $78\frac{2}{3}\times 170 = 13{,}373\frac{1}{3}$ lb. Ans.

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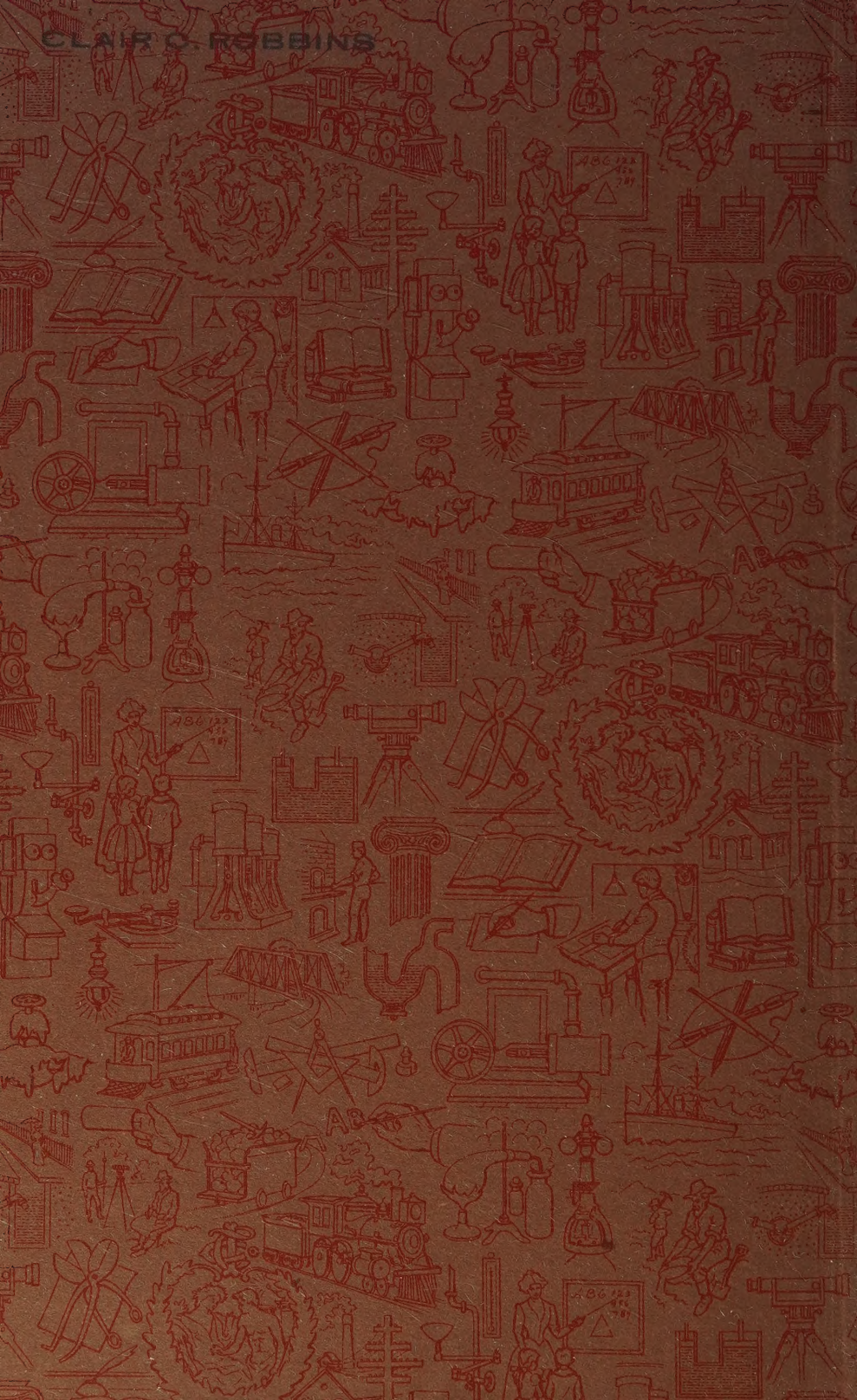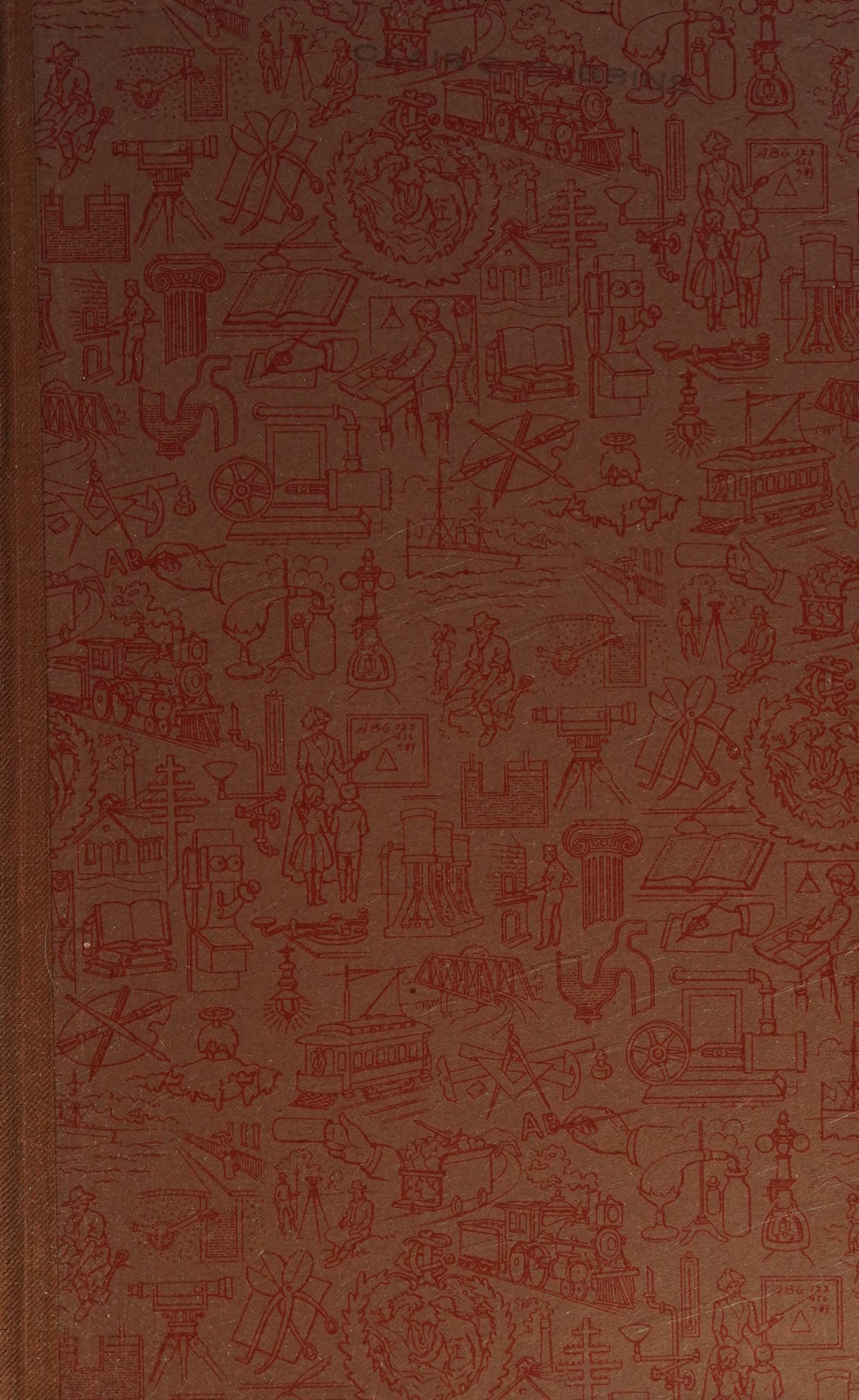
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